PSYC 2002
Introduction to statistics in psychology

Winter 2003

W-8

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Readings

- AA pp. 152-177 – Introduction to the *t*-test
- D pp. 137-149 – One-sample t-test; 159-163 – Paired sample *t*-test (related samples *t*-test)

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Lecture Outline: t-test

1. problems with *Z*-test
2. *t*-test as a solution to *Z*-test problems
3. degrees of freedom & *t*-test
4. probabilities for *t* distributions
5. hypothesis testing w/ *t*-test for single samples
6. one-tailed single sample *t*-tests
7. assumptions of single samples *t*-test

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Lecture Outline: t-test

8. related samples *t*-test (aka repeated measures & matched samples)
9. formula for related samples *t*-test
10. hypothesis testing with related samples *t*-test
11. using *t* in confidence intervals
12. assumptions underlying related samples *t*-test
13. *t*-test & power

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Problems with Z-test

<u>basic ideas underlying *Z*-test</u>

1.sample means approximate population means

2.standard error indicates how well a sample mean approximates a population mean

$$\sigma_M = \sigma / \sqrt{n}$$

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Problems with Z-test

3.the difference between the data and the hypothesis can be compared to the difference expected by chance; when the difference between the data and the hypothesis exceeds that expected by chance, the difference is said to be significant

$$Z = \frac{M_x - \mu}{\sigma_M}$$

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Problems with Z-test

problem

- *Z*-test procedure assumes more information is available then is typically available
- population standard deviation is not usually known, therefore it is impossible to calculate the standard error associated with a sample mean

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t-test as a solution to Z-test problems

simple solution to problems with *Z*-test

- use sample standard deviation to replace population standard deviation
- *Z*-test is now transformed into *t*-test

$$Z = \frac{M - \mu}{\sigma_M} \longrightarrow t = \frac{M - \mu}{S_M}$$

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t-test as a solution to Z-test problems

implications of using *S* instead of σ

- calculation of the sample standard deviation requires some modification from its population form
- the *SD* of a sample is a *biased estimate* of the population *SD*

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t-test as a solution to Z-test problems

- why is the *SD* of a sample biased?
- the sample's variance is slightly smaller than the population's variance
- therefore sample *SD* underestimates the population's variance

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t-test as a solution to Z-test problems

degrees of freedom

- *n*-1 is used to compensate for the underestimation of the sampling error in the calculation of the sample standard deviation

$$s = \sqrt{\frac{SS}{n}} \longrightarrow s = \sqrt{\frac{SS}{n-1}}$$

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t-test as a solution to Z-test problems

degrees of freedom

- sample mean is used to calculate the sample standard deviation
- use of the sample mean restricts the variability of the remaining scores
- only *n*-1 scores in the sample are free to vary once the sample mean is known

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t-test as a solution to Z-test problems

degrees of freedom

- if *n* = 2 and *M* = 6, then the sum of the scores in the sample must be 12
- if one of the scores = 8, then a restriction is placed on the identity of the other score – it must be 4
- in this example there is *n*-1 or 1 degree of freedom (*df*).

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t-test as a solution to Z-test problems

when to use *Z*-test vs. *t*-test

- if population standard deviation is known, use *Z*-test
- if population standard deviation is unknown, use *t*-test

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Degrees of freedom & t-test

degrees of freedom

- because the *t*-test uses sample standard deviation it must incorporate degrees of freedom into its usage
- as *df* increases, the better the sample standard deviation represents the population *SD*

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Degrees of freedom & t-test

- increasing *df* produces essentially the same effect as increasing *n*
- in turn, as the *df* increases, the more the *t*-statistic resembles the *Z*-statistic

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Degrees of freedom & t-test

t-distributions

- *t*-distribution is similar to normal distribution, especially for large sample sizes
- *t*-distribution is bell-shaped, symmetrical, and has a mean of zero

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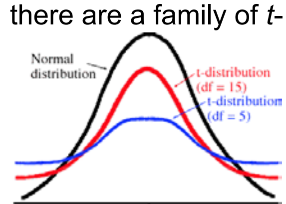
Degrees of freedom & t-test

t-distributions

- a separate *t*-distribution exists for every *df*. Thus, there are a family of *t*-distributions.



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Probabilities for t distributions

- probabilities associated with t-distributions are located in table form (A-2, p. 292)
- the probabilities in the table correspond to the cutoff sample *t*-value (i.e., $t_{critical}$) required to reject the null hypothesis

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Probabilities for t distributions

Cutoff Scores for the t Distribution

	One-Tailed Tests		
df	*.10*	*.05*	*.01*
1	3.078	6.314	31.881
2	1.886	2.920	6.965
3	1.638	2.353	4.541
4	1.533	2.132	3.747
5	1.476	2.015	3.365

- note – this is a subset of the *t*-distributions table for one-tailed tests

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Probabilities for t distributions

Cutoff Scores for the t Distribution

	One-Tailed Tests		
df	*.10*	*.05*	*.01*
1	3.078	6.314	31.881
2	1.886	2.920	6.965
3	1.638	2.353	4.541
4	1.533	2.132	3.747
5	1.476	2.015	3.365

- determine *df* & then look up appropriate *t*-value associated with alpha level and one- or two-tailed test

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Hypothesis testing w/ t-test for single samples

procedure similar to *Z*-test

1. formulate hypotheses (μ) and set alpha level
2. locate critical region as defined by alpha and *df* ($t_{critical}$)
3. collect sample data and calculate test statistic ($t_{obtained}$)
4. evaluate null hypothesis (compare $t_{critical}$ and $t_{obtained}$)

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Hypothesis testing w/ t-test for single samples: Example

- test the hypothesis that extra handling of infants leads to a change in growth so that by the age of two years children who have received extra handling will either weigh more or less than those children who do not receive extra handling

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Hypothesis testing w/ t-test for single samples: Example

- collect data from a sample of 20 two-year-old children, *M* = 31 lbs., *SS* = 16 lbs.
- assume that you know that the average population weight of two-year-old children is 26 lbs.

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Hypothesis testing w/ t-test for single samples: Example

step 1 - state hypothesis & select α

- H_0: $\mu_{\text{handling in infancy}}$ = 26 lbs.
- H_1: $\mu_{\text{handling in infancy}} \neq$ 26 lbs.
- α = .05 for two tails

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Hypothesis testing w/ t-test for single samples: Example

step 2 - determine critical region

- determine *df*
- n = 20; $df = n - 1 = 20 - 1 = 19$
- α = .05 (split between two tails)
- therefore critical region greater than t = +2.093 or less than t =-2.093

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Hypothesis testing w/ t-test for single samples: Example

step 3 - calculate test statistic

- M = 31 lbs.; SS = 16

$$S = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{16}{20-1}} = \sqrt{\frac{16}{19}} = \sqrt{.842} = .918$$

$$S_M = \frac{S}{\sqrt{n}} = \frac{.918}{\sqrt{20}} = \frac{.918}{4.472} = .205$$

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Hypothesis testing w/ t-test for single samples: Example

step 3 - calculate test statistic (continued)

$$t = \frac{M - \mu}{S_M} = \frac{31-26}{.205} = \frac{5}{.205} = 24.39$$

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Hypothesis testing w/ t-test for single samples: Example

step 4 - evaluate null hypothesis

- t_{obtained} 24.390 > t_{critical} 2.093
- test statistic falls within the critical region; reject null hypothesis
- handling makes a difference in the weight of children

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Hypothesis testing w/ t-test for single samples: Example

step 4 - evaluate null hypothesis

- write concluding statement
  - The mean weight of infants who were handled (M = 31 lbs.) was significantly different from the mean weight of the population of infants who were not handled (μ = 26 lbs.), $t(19) = 24.39$, $p < .05$.

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One-tailed t-tests: Example

- test the hypothesis that extra handling of infants leads to increased growth so that by the age of two years children who have received extra handling will weigh more than those children who do not receive extra handling

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One-tailed t-tests: Example

- collect data from a sample of 20 two-year-old children, *M* = 31 lbs., *SS* = 16 lbs.
- assume that you know that the average population weight of two-year-old children is 26 lbs.

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One-tailed t-tests: Example

step 1 - state hypothesis & select α

- H_0: $\mu_{\text{handling in infancy}} \leq 26$ lbs.
- H_1: $\mu_{\text{handling in infancy}} > 26$ lbs.
- α = .05 for one tail

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One-tailed t-tests: Example

step 2 - determine critical region

- determine *df*
- $n = 20$; df $= n - 1 = 20 - 1 = 19$
- α = .05 for one tail
- therefore critical region greater than $t = +1.729$

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One-tailed t-tests: Example

step 3 - calculate test statistic

- *M* = 31 lbs.; *SS* = 16
- $t_{obtained}$ is the same as in previous two-tailed example t = 24.390

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One-tailed t-tests: Example

step 4 - evaluate null hypothesis

- $t_{obtained}$ 24.390 > $t_{critical}$ value 1.729
- test statistic falls within the critical region; reject null hypothesis
- conclusion...handling increases the weight of children

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One-tailed t-tests: Example

step 4 - evaluate null hypothesis

- write concluding statement
  - The mean weight of infants who were handled (M = 31 lbs.) was significantly more than the mean weight of the population of infants who were not handled (μ = 26 lbs.), $t(19) = 24.39$, $p < .05$.

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Assumptions of single sample t-test

1. values in the sample must be independent observations
   - if each the value of each observation cannot be predicted from the value of another observation then the observations are independent

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Assumptions of single samples t-test

2. population sampled must be normal
   - *this is especially true if the sample size is small; however, for samples larger than n = 30, violations of the normality assumption have little effect on results*

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Related samples t-test (repeated measures & matched samples)

- an important research design involves testing the same individuals in two different conditions =*repeated measures design*
- similarly, individuals can be matched on some characteristic and then tested in two different conditions = *matched design*

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Related samples t-test (repeated measures & matched samples)

- both types of related samples (repeated measures & matched) designs have the advantage of controlling or limiting variability among participants, making them more sensitive to the effects of the independent variable

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Related samples t-test (repeated measures & matched samples)

- in both cases, a variation on the *t*-test formula called a *related samples t-test* (also known as *t-test for dependent means* or *matched samples t-test*) is used
- the related samples *t*-test differs from the single sample *t*-test in two ways
  1. it uses difference scores
  2. it assumes $\mu = 0$ ($\mu_1 - \mu_2 = 0$)

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General formula for related samples t-test

- difference scores

Person	Pre Treatment Score	Post Treatment Score	Difference
A	50	55	+1
B	49	50	+1
C	45	46	+1
D	57	56	-1
E	61	67	+6
F	60	60	0
		∑ =	+12

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General formula for related samples t-test

- mean difference score = 12/6 = +2
- sample difference scores are related to a population of difference scores

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General formula for related samples t-test

- begin with standard t-test formula

$$t = \frac{M - \mu}{s_M}$$

- by substitution the related samples formula is

$$t = \frac{M_D - \mu}{s_{M_D}}$$

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General formula for related samples t-test

- where the standard error of the differences is

$$s_{M_D} = \frac{s}{\sqrt{n}}$$

- degrees of freedom are defined as *n*-1 (the number of difference scores -1)

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Hypothesis testing with related samples t-test: Example

- Test the effects of a speed reading course by measuring the comprehension of readers skimming text for 15 minutes compared to their comprehension after taking a speed reading course.

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Hypothesis testing with related samples t-test: Example

Person	No Speed Reading	Speed Reading	Difference (After-Before)	Deviation	Squared Deviation
A	55	50	-5	-3	9
B	50	49	-1	+1	1
C	46	45	-1	+1	1
D	56	57	+1	+3	9
E	67	61	-6	-4	16
F	60	60	0	-2	4
		∑ =	-12	∑ =	40

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Hypothesis testing with related samples t-test: Example

step 1 - state hypothesis & select α

- H_0: $\mu_D = 0$.
- H_1: $\mu_D \neq 0$.
- α = .05 for one tail

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Hypothesis testing with related samples t-test: Example

step 2 - determine critical region

- determine *df*
- $n = 6$; df = 6 - 1 = 5
- α = .05 (split between two tails)
- therefore critical region greater than $t = \pm 2.571$

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Hypothesis testing with related samples t-test: Example

step 3 - calculate test statistic

- $M_D = \sum D/n = -12/6 = -2$
- $SS = 40$

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Hypothesis testing with related samples t-test: Example

step 3 - calculate test statistic

$$S = \sqrt{\frac{SS}{n-1}}$$

$$S = \sqrt{\frac{40}{6-1}} = \sqrt{8} = 2.83$$

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Hypothesis testing with related samples t-test

example

step 3 - calculate test statistic

$$S_{M_D} = \frac{S}{\sqrt{n}}$$

$$S_{M_D} = \frac{2.83}{\sqrt{6}} = \frac{2.83}{2.45} = 1.16$$

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Hypothesis testing with related samples t-test: Example

step 3 - calculate test statistic

$$t = \frac{M_D - \mu}{S_{M_D}}$$

$$t = \frac{-2-0}{1.16} = \frac{-2}{1.16} = -1.72$$

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Hypothesis testing with related samples t-test: Example

step 4 - evaluate null hypothesis

- $t_{obtained}$ -1.72 < $t_{critical}$ 2.571
- test statistic does not fall within the critical region; cannot reject null hypothesis
- conclusion…no significant effect of speed reading course

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Hypothesis testing with related samples t-test: Example

step 4 - evaluate null hypothesis

- write concluding statement
  - Readers' comprehension after taking a speed reading course (*M* = 53.7) did not significantly differ from their comprehension prior to taking the course (*M* = 55.7), $t(5) = 1.72$, $p > .05$.

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Hypothesis testing with related samples t-test: 1-tailed example

- use speed reading comprehension example; make explicit prediction that speed reading will result in increased comprehension compared to simply skimming
- changes from 2-tailed test
  - hypotheses
  - critical value for t

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Hypothesis testing with related samples t-test: 1-tailed example

- hypotheses must reflect the direction of change proposed as a result of the manipulation or treatment
- positive change – after > before
  - H_0: $\mu_D \leq 0$
  - H_1: $\mu_D > 0$
- negative change – before > after
  - H_0: $\mu_D \geq 0$
  - H_1: $\mu_D < 0$

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Hypothesis testing with related samples t-test: 1-tailed example

step 1 - state hypothesis & select α

- H_0: $\mu_D \leq 0$
- H_1: $\mu_D > 0$
  - why? speed reading is assumed to aid comprehension, therefore a positive difference should be observed, as reflected in the research hypothesis
- α = .05 for one tail

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Hypothesis testing with related samples t-test: 1-tailed example

step 2 - determine critical region

- determine *df*
- *n* = 6; df = 6 - 1 = 5
- α = .05
- therefore critical region greater than t = +2.015

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Hypothesis testing with related samples t-test: 1-tailed example

step 3 - calculate test statistic

- calculation of the test statistic is exactly the same as the calculation of the 2-tailed test statistic
- $t_{obtained}$ = -1.72

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Hypothesis testing with related samples t-test: 1-tailed example

step 4 - evaluate null hypothesis

- $t_{obtained}$ -1.72 < $t_{critical}$ 2.015
- test statistic does not fall within the critical region; cannot reject null hypothesis (note that $t_{obtained}$ is actually on the opposite tail from where it was hypothesized to be)
- conclusion…no significant effect of speed reading course

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Hypothesis testing with related samples t-test: 1-tailed example

step 4 - evaluate null hypothesis

- write concluding statement
  - Readers' comprehension after taking a speed reading course (*M* = 53.7) was not significantly better than their comprehension prior to taking the course (*M* = 55.7), *t*(5) = 1.72, *p* > .05.

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Confidence intervals using t

- confidence intervals using *t* are similar in form to those used with *Z*
  - parameter = *M* ± (α value) (SE)

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Confidence intervals using t

- single sample *t*-test
  - $\mu = M \pm ts_M$, where *M* is the sample mean, *t* is the *t*-value associated with the degrees of freedom for a sample size *n*, and *s* is the standard error of the mean
- related samples *t*-test
  - $\mu_D = M_D \pm ts_D$, where M_D is the sample mean difference, *t* is the *t*-value associated with the degrees of freedom for a sample size *n*, and s_D is the standard error of the mean difference

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Assumptions underlying related samples t-test

1. independent observations within each treatment condition
2. populations from which the difference scores are obtained must be normally distributed

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Power & the related samples t-test

- Table 8-7 indicates power for small, medium and large effect sizes in related *t*-tests for $\alpha = .05$
- related samples designs have more power than independent groups designs
- despite high power, useful to have control group for comparison

Next

- t-test for independent means
- AA pp. 178-198 – The *t*-test for independent means;
- D pp. 151-159 – *t*-test for independent samples