Social Welfare and Coercion in Public Finance

by

Stanley L. Winer [a] , George Tridimas [b] and Walter Hettich [c]

Revised September 3, 2008

Earlier versions of this paper were presented at Bar Ilan University 2006, the University of Eastern Piedmont 2006, the IIPF Congress, Paphos 2006, the National Tax Association, Boston 2006, Queen's University 2007, the World Congress of the Public Choice Societies, Amsterdam 2007, INFER Workshop, Cork, 2007, the University of Florida 2008 and Georgia State University 2008. We are indebted for comments by Jim Alm, Alberto Cassone, Thomas Dalton, Mario Fererro, Jon Hamilton, Arye Hillman, Larry Kenny, Sebastien Kessing, Carla Marchese, Fabio Padovano, Pierre Pestieau, Dan Usher, Jorge Vasquez and David Wildasin. Errors and omissions remain the responsibility of the authors.

[a]: School of Public Policy and Administration and Department of Economics, Carleton University, Ottawa, Canada K1S5B6. Tel: (613)520-2600 ext.2630. Fax: (613)520-2551. E-mail: stan_winer@carleton.ca;
[b]: School of Economics and Politics, University of Ulster, Shore Road, Newtownabbey, Co. Antrim BT37 0QB, UK Tel.: +44(0)28 90368273; Fax: +44 (0) 28 90 366847. E-mail: G.Tridimas@ulster.ac.uk;
[c]: Department of Economics, California State University, Fullerton, E-mail: whettich@fullerton.edu.

Winer's research is supported by the Canada Research Chair program. He is also indebted to the International Center for Economic Research, Turin, the Center for Economic Studies, Munich and POLIS at the University of Eastern Piedmont.

Abstract

This paper develops an expanded framework for social planning in which the existence of coercion is explicitly acknowledged. Key issues concern the precise definition of coercion for individuals and in the aggregate, its difference from redistribution, and its incorporation into normative analysis. We explore modifications to traditional rules for optimal fiscal policy in the presence of coercion constraints and determine the degree of coercion implied by traditional social planning. The paper maps the trade-off between social welfare and aggregate coercion and explores its implications for normative policy and the comparative evaluation of institutions, including competitive democracy.

Key words: Coercion, redistribution, social planning, optimal fiscal policy, marginal cost of funds, public goods, collective choice
JEL Categories: D70, H10, H20, H21

1. Introduction

Coercion is a fundamental and unavoidable aspect of public life. Although philosophers and constitutional experts have examined its nature at length, economists have not offered a fully integrated analysis of its role in economic life. In this paper, we introduce coercion into normative analysis by incorporating constraints that limit allowable coercion due to public policy choices. While the analysis applies generally to all types of policies, we study in detail coercion that arises whenever citizens experience a mismatch between what they receive from the public sector in the way of goods, services or transfers and what they pay in taxes.

Consider, for example, a group of people who have come together in a room for a common purpose and who must collectively set the temperature on a thermostat and then pay for the resulting use of energy. Inevitably, some will be too hot and some too cold, and even those for whom the temperature is just right may be unhappy with the resulting balance between what they pay and what they get.[1] Individuals can escape the situation if they move to another room or out of the building that represents the collectivity in this example. But if they stay, they must cope with the coercion implied by their assent to the collective decision.

The example embodies three essential aspects of coercion in the public economy. While the full topic of coercion is too extensive to be dealt with in a single paper, it will be useful to review all three aspects very briefly in order to provide a broader context for the discussion.

Individuals will voluntarily participate in a collectivity despite its coercive nature if joining makes them better off. This suggests a first focus, namely the analysis of why communities form, and under what circumstances people will join or leave them. A separate

[1] If Lindahl pricing was feasible and implemented, at *given* tax-prices everyone would vote for the same setting on the thermostat (or level of the public good). Disagreement over the setting of tax-prices may remain however.

body of work has developed on this topic.[2] A second focus deals with the choice of decision rules once a community has been formed. Here the classic work in a public finance context is by Wicksell (1896) and Lindahl (1919).[3]

Wicksell's proposal for approximate unanimity stems directly from his desire to minimize coercion exercised via the public finances for members of a community while providing for their welfare. Buchanan and Tullock (1962) made a further contribution by adding efficiency as a criterion in the choice of the decision rule and by considering the trade-off between decision costs and coercion associated with alternative rules, while Breton (1996) examined the relationship between coercion and budgetary institutions. One should note however that these authors did not provide a definition of coercion that could serve as a basis for welfare comparisons among different fiscal systems or institutional forms of government.

After collective decision rules have been put in place, participants in any community are inevitably faced with coercion arising from policies chosen in an externally fixed decision framework. This third aspect, which has received the least attention in the public finance literature so far, is the major focus of this paper. The usual approach in such a context has been to assume the existence of a planner who chooses public outputs and imposes taxes so as to maximize a social welfare function. In the example above, he or she would set the temperature in the room and fix tax rates for all the participants under the assumptions that there is agreement on the nature of the welfare function to be optimized, and that there is sufficient information to do so. While the analysis of specific policies, such as taxation, in the social welfare tradition has been extensive and highly successful, the existing literature has not so far dealt with the measurement and evaluation of coercion implicit in the possible actions of a planner.

---

[2] See, for example, Hirschman (1970), Skaperdas (1992), Usher (1993), and Alessina and Spolaore (2003).
[3] For interesting discussions of the Wicksell-Lindahl approach, see Escarrez (1967) and Dalton (1977).

In this paper, we examine the nature and measurement of coercion in a planning context by focusing on the extent to which individuals are unhappy with the balance between what they pay in taxes and what they get from the public sector. In section two we ask whether and how such coercion can be formally defined, why it is different from redistribution, and how it can be explicitly taken into account in the design of social plans by incorporating specially designed constraints into the planner's problem. We then proceed in section three to re-examine the well-known problem of linear income taxation with a public good if coercion constraints are imposed. (To provide additional generality, this aspect of the analysis is extended further in an appendix to include examination of the conditions for uniformity in a consumption tax in the presence of an optimal income tax.)

A special concern of fiscal analysis is the trade-off between social objectives. Introduction of coercion constraints allows us to formally explore the implications for social welfare of varying the degree of coercion in policy design. We pursue this topic in section four by considering the trade-off between social welfare and coercion, both in a general framework and in a more restricted model where a trade-off curve is explicitly constructed. Here the degree of coercion implied by traditional social planning is calculated.

Although the paper emphasizes policy analysis in a planning framework, it is important to realize that the use of coercion constraints also has applications in alternative institutional settings. Accordingly, in section four we also compare a social plan with the outcome of a competitive electoral system by locating both on the same welfare-coercion trade-off curve. The analysis suggests that electoral competition may lead society to a position on the downward sloping part of the trade-off, where social welfare is lower and the degree of coercion is higher than in the corresponding (coercion-unconstrained) social plan.

1.1 Some further considerations

Because the combination of social planning and a concern with coercion is unusual, it will be useful to consider the underlying ideas and literature somewhat further before beginning the formal analysis. In this regard, it should be noted that the design and implementation of constraints on the state has a long and distinguished history (see, for example, Gordon 1999 and Riker 1982). A concern with coercion has often arisen in the analysis of collective choice because individuals do not usually agree on the nature of the social objectives to be sought. For this reason at least, participation in communal affairs is often predicated on the preservation of rights that limit the scope of collective action. Concern with coercion also arises because of the desire to cope with the agency of politicians, bureaucrats or the military, and because of the possibility that some groups of citizens may coerce (or take advantage of) others using the collective choice process even in the absence of agency control problems. In this paper, we accept the premise that constraints on the ability of a collectivity to coerce individual citizens are desirable, and explore how such constraints *ought to* influence the structure of the public finances.

A precursor to such an inquiry in public finance can be found in the work of Simons (1938), who was concerned with establishing tax rules that limit interference in the lives of citizens and the private economy while also serving distributional ends. Buchanan and Congleton (1998, chap. 8) have more recently developed this approach further, calling for imposition of a very simple proportional tax system without a demogrant as a way to limit possible coercion. However, these authors do not provide a measure of the coercion implied by their proposals.

One should also note that coercion can also be imposed in other ways than through the balance between what a citizen gets and what they pay in taxes, including, for example, through

public administration (Alm, McClelland and Schulze 1992), conscription (Levy 1997), by regulating access to and limiting the scope of private markets (Wiseman 1989), as well as through the legal system (Anderson 2006, Leiser 2008). In this paper, we set administration and other, non-fiscal, dimensions of public policy aside, and confine the analysis to coercion arising from the balance between collectively provided goods or transfers received and taxes paid.

We realize that imposing constraints on a planner derived from a concern with coercion extends the analysis beyond criteria generally accepted in the planning literature. It also takes us outside of traditional public choice analysis. There have been other attempts to explicitly link collective choice concerns with the planning approach to policy analysis by imposing appropriate constraints. Acemoglu, Golosov and Tsyvinski (2008) have explored the nature of optimal fiscal policy rules when one acknowledges the existence of incentive compatibility constraints of politicians.[4] Here we deal with a different aspect of normative political economy - coercion of some citizens by others exercised through public policy - a problem that would arise even if somehow agency problems were absent.

In an earlier contribution that implicitly acknowledges the coercive aspects of collective fiscal choices, Boadway (2002) proposes to break the formulation of optimal policy into four stages: constitutional, legislative, implementation and market response. At each step the analyst takes the results of previous stages, which will include aspects of collective choice, as given in the design of a social plan. The approach in this paper is consistent with such a framework, but it goes further by explicitly incorporating coercion constraints into an analysis that could be used at

[4] They argue that optimal policy then requires the distortionary taxation of both capital and labour in order to reduce the amount of rents that need to be paid to the politician. One may note here that incentive compatibility constraints are themselves dependent on the nature of threats and force that is permitted in society. They are not given entirely by the state of nature. What is known about individual politicians or taxpayers depends on the range of actions that may be legitimately applied in uncovering individual characteristics and in preventing socially undesirable behaviour.

any relevant stage.

2. A Formal Definition of Coercion in Public Finance

We shall define coercion for an individual as the difference between this person's utility under what he or she regards as *appropriate* treatment by the public sector, and the utility that he or she actually enjoys as a result of social planning.[5] To make this definition concrete, it is necessary to explain what appropriate treatment means. In the public finance setting, there are two polar approaches to this issue, each corresponding to a particular view of the relationship between the individual and the state. The one we shall emphasize is illustrated in Figure 1.

[Figure 1 here]

In this '*individual-in-society*' approach, the counterfactual that must be part of a definition of coercion in the public finance setting is determined by finding the utility attainable if the individual could adjust the level of the public good at the given tax-prices he or she actually faces. To illustrate, we let the individual's actual tax-share be $\tau_j = (T_j/PG)$, where T_j is his or her total tax payment and P is the (assumed) constant supply price of the public good G, and assume, as in Buchanan (1968) and Breton (1974, 1996), that the individual believes he would pay this tax share if quantity adjustment were possible. The relevant tax rate for defining the counterfactual is $t_j = (\tau_j PG/Y_j)$ where Y_j is income and $\tau_j P$ is the individual's tax-price. The counterfactual level of utility then can be shown in Figure 1 as V^*_j where this person's utility is maximized subject to the tax-price line with slope $(\tau_j P/Y_j)$ and the corresponding counterfactual level of the public good is G_j^*. With V_j the actual indirect utility of individual j, coercion is then given in the figure by the utility differential $[V^*_j - V_j]$.

[5] Breton (1974) defines coercion in the public finance context as depending on the deviation of marginal evaluations of public services from tax-prices. While (the total amount of) coercion as defined below varies with this difference, as we show below, it is not coercion itself.

Formally, in the *individual-in-society* approach, an individual's coercion is defined as

$$[V^*_j(G^*_j, W_j, \tau_j P) - V_j], \text{ where } G^*_j = \arg\max_{\{G\}} V_j(G, W_j, \tau_j P) \quad (1)$$

and where W_j is the person's wage or ability. This definition is implicit in the work of Wicksell, Lindahl, Buchanan and Breton. Here it is assumed that each person accepts some coercion by society, along with a socially determined tax-price.[6] In terms of our original example, the individual remains in the room, submitting to (but critically judging) the outcome of the choice process.

It should be noted that the definition of coercion in (1) implies that it is simultaneously determined along with the parameters of the fiscal system. This holds because tax-prices and the level of the public good partly determine coercion while, in turn, the degree of coercion will be taken into account in deciding upon the fiscal system and its implied tax-prices. This simultaneity makes the investigation of coercion-constrained fiscal systems more complicated, but also more interesting than if the counterfactual was specified independently of contemporaneous policy choices.

Before we can specify coercion constraints that are to be imposed on the planner, there are two additional distinctions to consider: First, coercion can be determined on an individual basis as implied by the preceding discussion, or it can also be defined for a group. While applying constraints to each individual is consistent with the tradition initiated by Wicksell and Lindahl, we also want to explore a group approach that allows for stronger policy judgments and a greater degree of coercion. Although there isn't a complete parallel, defining coercion over a

[6] Hart (1961) argues, as we see it, that to accept this coercion, the individual must be assured that law abiding citizens will not be taken advantage of. Or, in other words, that the situation we are describing must involve the threat of legal sanction backed by force to deal with possible tax evasion. The definition of coercion here and that in the legal tradition may thus be seen as having a common basis. To proceed with integration of coercion into social planning, however, it is instrumental to continue with our definition, which one could also describe as being derived from coercion in the legal sense. On coercion in the legal tradition, see also Anderson (2006).

group of individuals is similar to the use of the Hicks-Kaldor potential compensation criterion.

As regards the second distinction, coercion can be defined either by using utility or by approximating changes in utility levels by using levels of the public good (or, policy outcomes of a particular nature), a method that follows Buchanan and Breton and that proves useful for working out examples. The resulting four possibilities are summarized as coercion constraints in Table 1, where we use K_j to denote the 'degree' of coercion applied to individual j (and, later, κ for the associated shadow price) because the Greek word for coercion is *katanagasmos.*

[Table 1 here]

The constraints are specified in the table as inequalities. However in the analysis below, we assume for mathematical convenience that coercion is applied up to the maximum allowed, so that in practice we deal only with equalities.

Table 1 is based on the *individual-in-society* approach to the counterfactual. There is a second, polar choice that could be made, namely to think of individuals as judging social outcomes from a perspective in which they alone decide what is best for them *and* for others. In this *individual-as-dictator* approach (not shown in Figure 1), the counterfactual utility is determined by maximizing the person's indirect utility subject to the government budget restraint that shows all feasible combinations of tax rates and actual public good levels. Because the counterfactual then involves a choice directly from the government budget constraint, it does not depend on what the planner does. The *individual-as-dictator* approach is more appropriate in studying situations where individuals consider whether to join a group rather than for the third aspect of the coercion problem, in which a community is already assumed to have formed. For this reason, we comment only briefly on the implications of adopting the *individual-as-dictator* approach.

2.1 Coercion approximated by levels of the public good

Table 1 indicates that coercion can be approximated using the level of the public good, either on an individual or on an aggregate basis. The argument is illustrated in Figure 2. Because the marginal evaluation of the public good declines with the size of the public sector, the difference in utility in (1) above is monotonically related to the difference between the level of the public good in the counterfactual and that provided by the planner, *(G* - G).*

[Figure 2 here]

To allow for cases in which the difference between *G* and *G** is sometimes negative and sometimes positive, we shall use the square of the difference in public good levels as an index of coercion.[7] Thus people who want less of the public good in the counterfactual (illustrated in Figure 2) are treated symmetrically with those who would like more. This is so because in either case, the actual utility at given tax-prices must be less than what it would be in the individual's preferred counterfactual

2.2 Coercion versus redistribution

It is useful at this point to ask why maximization of social welfare does not already deal with coercion as defined above, even though the difference between benefits and costs for each individual is reflected in individual (indirect) utilities and therefore in the social objective. The reason lies in the fact that the social planning approach posits no limits on the loss or gain in utility for particular individuals or groups occurring as part of a social plan. Any amount of redistribution required in the course of maximizing social welfare is implicitly regarded as acceptable, that is, as a matter of social solidarity with the planner's objective. We shall return to

[7] It should be noted that it may be the case that only one of these two types of individuals will arise in a fiscal system. The reason is that coercion depends on what individuals *think* is appropriate treatment *for themselves*, not what is actually feasible for society as a whole.

the difference between coercion and redistribution in section four where the coercion-constrained social planner's optimization problem is formally compared to a traditional plan.

It might be argued that application of the Pareto criterion - that only reallocations leaving every one better off are permissible - can attenuate concern with coercion. Strict application of the Pareto criterion limits the degree of individual coercion for moves from the status quo. It does not, however, alleviate any mismatch between benefits received and taxes paid that is embedded in the status quo itself. Moreover, much applied work using social welfare analysis goes beyond the strict Pareto criterion, which is too weak to allow for most social action, using the Hicks-Kaldor potential compensation criterion instead. In that case, reallocations are considered desirable even if some people become worse off, as long as gainers could *in principle* more than compensate losers. For this reason, an explicit concern with coercion is justified and needed in most practical instances.

To see in general terms that coercion in public life is widely viewed as distinct from income redistribution, it is also instructive to consider the Bill of Rights in the United States and similar documents or unwritten constitutional rules in other countries. The rights afforded by these documents are intended to apply equally to the poor and rich; they were not created with reference to income levels, but with reference to individual lives. There may, of course, be an interaction of redistribution as traditionally defined and coercion, but this only reinforces the insight that redistribution is not the sole origin of coercion. [8] A similar point is emphasized by Wicksell (1896), who reminds us that an imperfect correspondence between what people pay in

[8] In an interesting paper complementary to the current one - the perspective is that of the first issue identified above - Perroni and Scharf (2003) develop a positive theory of the self-enforcing fiscal system. The problem they begin with is that there is no external power to enforce the power to tax so that ultimately, in their view, all fiscal systems must be self-enforcing equilibria where the continual consent of the public is sought. They search for efficient, self-enforcing equilibria that are robust to renegotiation among groups of citizens. As a consequence, they claim (result 4) that when citizens have identical preferences, efficiency and renegotiation proofness requires horizontal equity in taxation. But, as they state, this is "fully unrelated to any distributional goal" (p. 9). Rather, in their approach, it is a matter of insuring the stability and viability of society as a whole.

taxes and what they receive in a democratic society providing public services and collecting taxes differs from voluntary redistribution and should be a cause for concern.[9]

2.3 Using constraints to model the role of coercion

To develop a normative approach that allows us to compare and evaluate specific fiscal policies and electoral mechanism, we proceed by imposing coercion constraints on maximization of social welfare as usually defined. The use of constraints in this way may be defended on both conceptual and practical grounds.

We have already noted the long history of attempts to limit the power of politicians and coercion of some citizens by others through the fiscal system. The most important way in which limits on such activities have been introduced into political arrangements is by written or unwritten constitutional provisions restricting the power of government to abridge individual rights. Such provisions do not in principle allow for a trade-off between the rights that are given and other policy objectives. They may, of course, be subject to interpretation by the courts, but always with the understanding that the rights take precedence over other public aims. The setting of boundaries or constraints on public action thus represents a well-known and tested approach to dealing with coercion in public life.

A strict welfarist might argue that if it matters to an individual that he is being coerced, then this should be reflected in his or her utility function. And if it is, then social welfare maximization will take this concern into account. However, introducing coercion into a utility function is obviously a shorthand for a complicated social situation.[10] It is hardly clear that this is the best way to proceed, even if using constraints appears to involve the introduction of a non-

[9] "From the point of view of general solidarity...parties and social classes should...share an expense from which they receive no great or direct benefit. Give and take is a firm foundation of lasting friendship... It is quite a different matter, however, to be forced so to contribute." Wicksell (1896/1958, 90).

[10] The same is true if coercion is put into the welfare function but outside of individual utilities.

welfarist criterion.

Consider an analogy to modeling the social role of money. Macroeconomists have tried to come to grips with the role of money in society either by putting money into the utility function following Patinkin (1965) - an obvious approximation to the complex social role of money - or by adding constraints to the specification of the economy while continuing to model individual economic agents in more or less traditional fashion (e.g. the cash-in-advance constraints of Clower 1967). Our approach is analogous to the second method. We add coercion constraints to a planning problem in order to incorporate an important aspect of collective choice in a simple and (we think) revealing manner. While the justification based on the actual way in which boundaries on collective action are actually set seems to us sound, to an extent the approach we develop is also based on our judgment that it is a useful way to proceed in an important area of research where little progress has so far been made. Our investigation remains essentially welfarist in intention, though in a broader context where a concern with limits on the degree of coercion is regarded as important for the social health of the community.[11]

In what follows, we mainly employ the definition of coercion that is analogous to the application of Hicks-Kaldor potential compensation. The corresponding analysis with individual constraints is provided in an Appendix. One may also want to acknowledge that coercion may serve as a method of reducing the costs of policy actions, such as that associated with the excess burden of taxation, thus having a productive as well as a harmful social role. We do not incorporate this possibility into the analysis. Excess burdens are defined in the usual manner, independently of the degree of coercion.[12] The emphasis here is on defining coercion arising

[11] Kaplow (2007) also points out in his review of Pareto and competing principles that formal methods that depart from standard social welfare maximization may still be essentially welfarist in nature.

[12] For example, we do not explicitly allow the planner to force independent evaluations of ability on taxpayers, or to coercively uncover economic activity, thereby relaxing incentive compatibility constraints.

from the collective provision and financing of public goods and services, and on investigating how acknowledging the limits to such coercion ought to alter the structure of optimal policy.

3. Coercion-Constrained Optimal Linear Income Taxation with a Public Good

Having offered a formal definition of coercion, we can now show how the introduction of coercion constraints alters the welfare analysis of a fiscal system in which a pure public good is financed with a linear income tax of the form $T_j = tY_j - a$.[13] We choose this application because it permits easy comparison with accepted results in the literature. For the same reason, and because nothing privileges the income tax in dealing with coercion, we demonstrate in the Appendix how the analyses of Atkinson and Stiglitz (1976, 1980) concerning the indirect - direct tax mix and the conditions under which uniformity of consumption taxation is optimal are substantially amended by incorporation of a concern with coercion.

This investigation differs from that of earlier pioneers Simons (1938) and Buchanan and Congleton (1998). In one sense it is less general: they derive the nature of a fiscal system they regard as most efficient given the satisfaction of their concern to limit coercion. This is the genesis of Simons' advocacy of the broad base income tax, which by its breath prevents governments from 'dipping into great incomes with a sieve'. By starting with a fiscal system of a particular type, we cannot replicate this sort of investigation. On the other hand, we shall be able to proceed with greater analytical depth with regard to the definition of coercion and its role in determining parameters of the fiscal system, allowing careful comparison of the coercion-constrained system with the traditional social plan as well as an investigation of the trade-off

[13] Here neither the tax rate t nor the lump sum component a varies across individuals, thus providing a simple way of introducing the excess burden of taxation while also ruling out a Lindahl voluntary exchange equilibrium in which taxes are raised without any welfare loss. With $a = 0$ the tax is proportional to income, with $a > 0$ it is progressive, and with $a < 0$ regressive. For comparability with the literature, we follow Sandmo's (1998) notation.

between coercion and social welfare.

Assume then that there are N individuals indexed by j, each maximizing utility defined over a private good X_j, leisure L_j and a public good, and receiving a fixed wage W_j . The individual's optimization problem is:

$$Max\ U_j = U_j(X_j, L_j, G) \quad subject\ to \quad X_j = (1-t)W_j(1-L_j) + a; \quad j = 1, \ldots, N \tag{2}$$

where, in addition to previous definitions, H_j is the supply of labour, with $L_j + H_j = 1$.[14]

To establish the counterfactual, we consider the individual when he is free to choose the level of the public good G_j, given his (average) tax share τ_j , which is assumed by the individual to be constant with respect to the level of the public good. This tax share is given by the ratio of the tax paid by j to total tax revenue; $\tau_j = T_j / \Sigma_j T_j$. We note for later use that with the linear tax system this tax share is

$$\tau_j = \frac{t\, Y_j - a}{t\, \Sigma_j Y_j - Na}. \tag{3}$$

Since the marginal cost of the public good P is constant, the actual tax price per unit of G, $\tau_j P$ is also the one that applies to marginal changes in public services when viewed from the perspective of each individual. The individual's optimization problem we can use to define his or her counterfactual then can be stated as

$$\underset{\{G, X, L\}}{Max} \quad U_j = U_j(X_j, L_j, G_j) \quad subject\ to \quad X_j + \tau_j P G_j = W_j(1-L_j). \tag{4}$$

Solving the first order conditions, $U_{jX} = \lambda^*_j$, $U_{jL} = \lambda^*_j W_j$ and $U_{jG} = \lambda^*_j \tau_j P$, yields the

[14] To help the reader follow later derivations, we note here that solving this problem yields the usual condition $U_{jL}/U_{jX} = (1-t)W_j$, the final demand for the private good $X_j = X_j[(1-t)W_j, a, G]$, the labor supply $H_j = H_j[(1-t)W_j, a, G]$, and the indirect utility function $V_j = V_j[(1-t)W_j, a, G]$. Denoting the marginal utility of income by λ_j, the partial derivatives of utility with respect to the fiscal variables for person j are $V_{jt} = -\lambda_j Y_j$, *with* $Y_j = W_j(1-L_j)$; $V_{ja} = -\lambda_j$ and $V_{jG} = U_{jG}$., and the marginal willingness to pay for the public good is $m_j = U_{jG} / U_{jX} = V_{jG} / \lambda_j$.

counterfactual indirect utility V^*_j in (1), where the (*) reflects the fact that the individual is considered to be choosing his most preferred level of G at the *given* tax-price.

3.2 Social welfare maximization under aggregate coercion

In choosing fiscal policy instruments, the coercion-constrained planner chooses *G, t* and *a* to solve the problem of maximizing social welfare subject to a budget restraint:

$$Max \quad S = \sum_j V_j \quad subject\ to \quad t\sum_j W_j H_j - Na = PG. \tag{5}$$

In addition, the planner faces one or more coercion constraints. In this respect, we begin for simplicity of exposition with case 2 in Table 1, where coercion is defined using utility levels and aggregated across individuals. As we have already pointed out, this case is analogous to the use of the Hicks-Kaldor criterion in cost-benefit analysis. The corresponding Lagrangean for the constrained social planning problem is:

$$\mathcal{L} = \sum_j V_j + \mu [\, t\sum_j W_j H_j - Na - PG \,] + \kappa [\, K - \sum_j (V^*_j - V_j \,)]. \tag{6}$$

In solving this problem, V^*_j and the shadow price of coercion κ are determined simultaneously along with policy instruments, so the coercion constraint cannot be simply collapsed into a part (V_j) that can be added to the social welfare function, and a remainder that is constant and so can be ignored. In other words, acknowledging coercion does not amount to simply placing added weight on the utility of some individuals in a social plan. Concern with coercion requires that weight be given to the *counterfactual* level of utility for each individual V^*_j.[15]

By the envelope theorem, the shadow price of coercion κ is equal to dS^*/dK where the star denotes an optimal value. Social welfare will reach its maximum when this derivative is

[15] One may also note that if there is only one person, or if everyone is identical, there will be no difference between V^* and V at an optimum, and any coercion constraint will be irrelevant. Coercion as defined here has no meaning in a single agent planning model.

zero, at a corresponding and generally non-zero level of K (to be derived in the next section for a particular case). Welfare will then be equal to what it would be in the unconstrained or traditional social plan. Accordingly, we may say that when the shadow price is positive and high, there will be a large payoff (in terms of social welfare) from solidarity with the aims of the unconstrained social planner. We shall consider the shadow price of coercion further in what follows, noting at this point only that the formulation of the coercion constraint does not insure that it will always be positive, because K may exceed the level of coercion that is consistent with the traditional social welfare optimum.

Using the definition of V^*_j, we have the first order conditions for the coercion - constrained planner's problem:

$$(1+\kappa)\textstyle\sum_j \lambda_j W_j H_j = \mu\,[\sum_j W_j H_j + t\sum_j W_j(\partial H_j/\partial t)] + \kappa \sum_j \lambda^*_j P G^*_j \;\boldsymbol{(\partial \tau_j/\partial t)} \quad (7.1)$$

$$(1+\kappa)\textstyle\sum_j \lambda_j = \mu\,[N - t\sum_j W_j(\partial H_j/\partial a)] - \kappa\sum_j \boldsymbol{\lambda^*_j PG^*_j\;(\partial \tau_j/\partial a)} \quad (7.2)$$

$$(1+\kappa)\textstyle\sum_j \lambda_j m_j = \mu\,[P - t\sum_j W_j(\partial H_j/\partial G)]. \quad (7.3)$$

where m_j is the marginal rate of substitution between public and private goods for person j, and λ_j is his or her marginal utility of money. These equations feature two important new elements that are absent from traditional optimal taxation but that are always present in the analysis of coercion: (a) the translation of tax structure into the tax price – appearing above as $(\partial \tau_j/\partial t)$ and $(\partial \tau_j/\partial a)$; and (b) the translation of the tax price into welfare via the demand for G - shown as $\lambda^*_j P G^*_j$.

To explore the implications of these equations for fiscal structure, we first use them to characterize optimal coercion-constrained policy in a general manner. We begin with (7.3) which characterizes the coercion-constrained size of government. Define the covariance between the λ_j and m_j by $\sigma^2_{\lambda m} = (\sum_j \lambda_j m_j /N) - (\sum_j \lambda_j /N)(\sum_j m_j /N)$. Also define the means $\lambda = \sum_j \lambda_j /N$ and $m = \sum_j m_j /N$, and the normalized covariance between λ_j and m_j, $\delta = \sigma^2_{\lambda m}/\lambda m$. The latter reflects the distributional characteristics of the public good.

Then substituting these definitions into (7.3) and manipulating yields a characterization of the optimal coercion-constrained level of the public good:

$$\left(\Sigma_j m_j\right)(1+\delta)(1+\kappa) = \frac{\mu}{\lambda}\left(P - t\Sigma_j W_j \frac{\partial H_j}{\partial G}\right). \tag{8}$$

Equation (8) is a generalization of the Samuelson condition as amended by Atkinson and Stern (1974) to acknowledge a concern with coercion of some citizens by others through the fiscal system. The right hand side is the familiar (coercion-unadjusted) social marginal cost of the public good net of the induced revenue effects of public provision on labour supply.[16]

The left-hand-side represents the social marginal benefit from public provision in the presence of the coercion constraint. The first two terms here are also familiar: the sum of the marginal rates of substitution between private and public goods $\sum_j m_j$; and the term $1+\delta$ which adjusts the marginal rates of substitution for the distributional characteristics of public good. The new term $(1+\kappa)$ reflects the effect of the coercion constraint, and combines with $(1+\delta)$ to represent the average effect of coercion. The traditional planning solution in the absence of a coercion constraint is derived by setting $\kappa = 0$ in (8). Comparison of the implications for fiscal structure of (8) with those of the traditional formula is not straightforward however, as the solution depends on the level of coercion as well as on the relationship between K and its shadow price, which is endogenous. We discuss the $\kappa - K$ relationship and the welfare-coercion trade-off in the next section**.** Here we note that if both δ and κ are positive, (8) indicates that higher values of κ will tend to be associated with larger public sectors.[17]

Condition (8) also shows that the coercion-adjusted marginal cost of funds (MCF)

---

[16] It is equal to the product of the marginal valuation of government revenue, μ/λ and the net (of induced revenue) rate of transformation of the public good, $P - t\sum_j W_j(\partial H_j/\partial G)$.

[17] To preserve (8) when κ is large, $\sum_j m_j$ will tend to be small.

appropriate for policy analysis is $\mu/[\lambda(1+\kappa)]$. This coercion-adjusted MCF will tend to be low when κ is high, that is, when increasing K to relax the coercion constraint has a large payoff in terms of social welfare. Again the traditional formula is derived simply by setting the shadow price to zero. And while comparison with the two formulas is complicated by the endogeneity of κ, it should be noted note that the MCF remains relevant in the present context as an analytical concept.

3.2.1 The optimal coercion - constrained tax rate

To derive the optimal income tax rate in the presence of coercion, we proceed as follows. Multiplying (7.1) by (1/N) and (7.2) by $(\sum_j W_j H_j/N^2)$, subtracting the latter from the former, and using the Slutsky decomposition, $\partial H_j/\partial t = s_j - W_j H_j(\partial H_j/\partial a)$, yields

$$(1+\kappa)\sigma^2_{\lambda Y} = t\frac{\mu}{N}\left[\Sigma_j W_j s_j - \sigma^2_{Ya}\right] + \frac{\kappa}{N}\Sigma_j \lambda^*_j PG^*_j\left(\frac{\partial \tau_j}{\partial t} + \frac{\partial \tau_j}{\partial a}\bar{Y}\right) \tag{9}$$

where $Y_j = H_j W_j$, and the negative covariance $\sigma^2_{\lambda Y}$ shows the relationship between the marginal utility of income and income from work and reflects the distributional effects of income taxation. $\sigma^2_{\lambda Y}$ is negative since the higher the level of income, the lower is the marginal utility. The term $\overline{WS} = \Sigma_j W_j s_j / N$ is the mean substitution effect of taxation on labor supply, which is also negative.

The covariance term in (9),

$$\sigma^2_{Ya} = \frac{1}{N}\left[\Sigma_j W_j \frac{\partial H_j}{\partial a} W_j H_j - \left(\Sigma_j W_j \frac{\partial H_j}{\partial a}\right)\left(\frac{\Sigma_j W_j H_j}{N}\right)\right],$$

shows the relationship between income and the income effect of taxation; it is non-negative when the effect of income on labor supply is small for those with high incomes. The quantity $q_j = \partial \tau_j/\partial t + (\partial \tau_j/\partial a)\bar{Y}$ in (9) is the change in the tax share of j when the tax rate and the lump-sum transfer both change, where a bar above Y denotes its mean. If we let $\psi_j \equiv V^*_{j\tau} = -\lambda^*_j PG^*_j$, we can write $\Sigma_j \lambda^*_j PG^*_j [\partial \tau_j/\partial t + (\partial \tau_j/\partial a)\bar{Y}] = \Sigma_j \psi_j q_j$. Using the covariance formula, the right side of this last

expression is $\Sigma_j \psi_j q_j = N\sigma^2_{\psi q} + N\overline{\psi}\overline{q}$, where $\overline{\psi}$ and $\overline{q}$ denote the mean values of ψ_j and q_j respectively. Here $\sigma^2_{\psi q}$ captures the relationship between the marginal utility of the tax share and the marginal tax share, and is an important determinant of how coercion is spread across the community. The value of $\sigma^2_{\psi q}$ depends on the size of the parameters of the utility function and is therefore an empirical matter. If tax payers who experience a large increase in their tax shares will also experience a significant fall in utility, $\sigma^2_{\psi q}$ will be negative. Also, by differentiating the tax share we obtain $\Sigma_j \partial\tau_j/\partial t = \Sigma_j \partial\tau_j/\partial a \bar{Y} = 0$ and thus $\overline{q} = 0$.[18]

This last result and the definition of $\sigma^2_{\psi q}$ in (9) leads to the coercion-constrained optimal income tax rate:

$$t = \frac{(1+\kappa)\sigma^2_{\lambda Y} + \kappa\sigma^2_{\psi q}}{\mu\left(\overline{WS} - \sigma^2_{Ya}\right)}. \tag{10}$$

When the shadow price is positive and the $\sigma^2_{\psi q}$ covariance is negative, the optimal rate t rises with κ since the $\sigma^2_{\lambda Y}$ covariance is negative and so is the denominator. That is, the higher is the payoff to solidarity with the aims of the unconstrained social planner, the higher the optimal coercion-constrained income tax rate. But note that the size of government and of tax rates may still be lower in the constrained situation than in the traditional social plan. [19]

By comparing equation (10) with its traditional counterpart, obtained by setting $\kappa = 0$ (which need not be stated explicitly here), one can see that the more general formulation of the optimal tax rate features four new terms in comparison to the standard formula: the shadow price

[18] To derive this expression, we differentiate τ_j in (3) with respect to t and a, and recognize that a change in t and a affects the level of income.

[19] If the *individual-as-dictator* counterfactual had been used to define coercion, the counterfactual utility would no longer depend on the choice of the fiscal system, and the resulting formula for the tax rate will be simpler, omitting the second term in the numerator of (10). (The general form of the solution for G given by equation (8) is the same with both counterfactuals, though the level of G will differ in each case.)

of coercion κ; the covariance $\sigma^2_{\psi q}$ between the marginal utility of the tax share and the marginal tax share; and via $\sigma^2_{\lambda Y}$, the translation of the tax system into tax shares $\partial \tau_j / \partial a$ and the translation of the tax price into welfare via the demand for G, $\lambda^*_j P G^*_j$.[20]

The analogue to conditions (8) and (10) when individual coercion constraints are imposed on the planner is worked out in the Appendix. The solutions are much more complicated, involving also the distributional pattern of coercion as one should expect. It turns out to be the case that in comparison to the Hicks-Kaldor like situation, government size and tax rates may be larger or smaller when coercion is specified at the individual level.

Since nothing privileges the income tax in dealing with social welfare and coercion, in the Appendix we also provide a characterization of the optimal, coercion-constrained fiscal system when both a consumption tax and an income tax are employed. For comparison with the existing literature, this demonstration starts with the Atkinson/Stiglitz (1976, 1980) conditions for uniformity of the consumption tax in the presence of an optimal linear income tax. We show that formally introducing a concern with coercion invalidates this theorem, essentially because consumption taxes, and hence the balance between what individuals pay and what they get from the public sector, depends in part on who consumes what. Social welfare is always higher, given an aggregate coercion constraint, when a consumption tax appropriately discriminates among citizens according to the pattern of consumption.

---

[20] Concerning the lump sum tax component 'a', the following general rule is offered, one that des not appear to have a counterpart in the optimal tax literature. Upon rearranging (7.2) we have that $\sum_j \left((1 / MCF) - \partial H_j / \partial a \right) / N = 1 - \kappa \{ \sum_j (\lambda_j * PG * / \mu_j) \bullet \partial \tau_j / \partial a) \} / N$. The left-hand-side shows the average net social marginal valuation of income in terms of government revenue which is coercively extracted. The right-hand-side shows the value of $1 of government expenditure corrected for the fact that it is coercively spent by the government rather than freely by the individual. For optimality the two must be equal.

4. The Trade-off between Social Welfare and Coercion

The existence of a trade-off between welfare and coercion in the present framework allows us to explore some of the implications for public finance of the clash of social objectives with individual rights embedded in the coercion constraints.

It is helpful to begin exploration of the trade-off by considering the relationship between the shadow price of coercion and the aggregate degree of coercion in the case of the linear income tax. In this case, maximization of social welfare subject to the government budget and an aggregate coercion constraint involves solving a system of five equations: first order conditions (7.1) – (7.3), the government budget constraint $t\sum_j W_j H_j = Na + PG$ and the coercion constraint $\sum j\ (V^*_j - V_j) = K$. The five unknowns are the three fiscal parameters, t, a and G, and the two Lagrange multipliers. This means that the solution for the shadow price of coercion (along with the rest of the endogenous variables) is a function of the distribution of individual tastes for work, leisure and consumption, captured by the parameters of the utility function and denoted by Γ; the characteristics of the distribution of earning abilities captured by the wage rates and denoted by W; the marginal cost of the public good P; and the level of coercion K.

We may write the implied solution for the shadow price of coercion (as well as that for all other endogenous variables) as $\kappa = \kappa(\Gamma, W, P, K)$, and the general solution for coercion-constrained social welfare as $S^* = F(\Gamma, W, P, K)$. As noted earlier, these are linked by the envelope theorem $dS^*/dK \equiv \Sigma_j (dV_j / dK) = \kappa$. If κ is positive (negative), coercion-constrained social welfare is rising (falling) with the level of coercion K, and when κ is zero, social welfare is at its maximum and policy instruments will conform to their traditional optimal tax values. $\kappa(\Gamma, W, P, K) = 0$ thus defines implicitly the value of K (K^{OT}) that is implied by traditional social planning.

Figure 3 illustrates one possible relationship between the shadow price and the aggregate

degree of coercion. The part of the κ - K curve labeled the 'consensual society' is where welfare and coercion are positively correlated. This part is where any society which positively values both social welfare and the absence of coercion would like to be. Such a society would not want to be at the point associated with a traditional social plan, where $K = K^{OT}$. On the part of the $\kappa - K$ curve to the right of K^{OT}, welfare increases as coercion declines. This part is to be avoided without compensating virtues.

[Figure 3 here]

In the example shown, the implied trade-off between social welfare and coercion would be concave with its peak at K^{OT}. But this is a hypothetical relationship. What does the welfare - coercion trade-off look like for the linear income tax case we have explored above? This turns out to be a difficult question to answer. As shown in the Appendix, in the linear income tax model with an aggregate coercion constraint, we cannot sign the slope of the shadow price – coercion relationship, the derivative $d\kappa/dK$, without further assumptions.

In the next section, we provide an explicit derivation of the trade-off between social welfare and the degree of coercion in a more simplified setting. Here we shall also compare the point on the trade-off chosen by a traditional social planner with the outcome that results from the operation of an exogenously determined collective choice process.

4.1 The welfare – coercion trade-off in a simplified setting

In order to proceed further, we assume that taxation is strictly proportional to income, utility is Cobb-Douglas, and that aggregate coercion is measured using levels of the public good as in case 4 of Table 1. The utility function of voter j is defined over private consumption, X_j and a public good G; $U_j = \alpha_j lnX_j + \gamma_j lnG$, $\alpha_j+\gamma_j=1$. It is further assumed that there are N individuals in the society, $j = 1,\ldots,N$.

Income Y_j is assumed to be exogenous. Since the tax system is proportional at rate t, for each citizen, $X_j = (1-t)Y_j$. Normalizing the unit price of the public good to unity, the budget constraint of the government is $t\Sigma_j Y_j = G$.

Aggregate coercion will be defined by the level of the public good. The coercion constraint then is

$$\Sigma_j(G - G^*_j)^2 = K, \quad j = 1, \ldots, N. \tag{11}$$

Because there are only two policy instruments linked by the government budget restraint in the simplified setting, this constraint is sufficient to determine the level of the public good in the coercion-constrained planning problem. (In the situation involving both a consumption and an income tax, considered in the Appendix, this is not so.)

To define the counterfactual level of the public good G^*_j, let citizen j be free to choose the level that he or she would like given his or her tax share τ_j. The relevant problem is to maximize U_j subject to the budget constraint $Y_j = X_j + \tau_j G_j$, where τ_j is the ratio of the tax paid by j to the total tax revenue, $\tau_j = T_j / \Sigma_j T_j$. With a proportional income tax system, the latter is simply $\tau_j = Y_j / \Sigma_j Y_j$. Indirect utility of j can then be written as $V_j = (1-\gamma_j) ln\{[1-(G_j/\Sigma_j Y_j)]Y_j\} + \gamma_j \ln G$, and maximization of this with respect to G_j yields

$$G^*_j = \gamma_j \Sigma_j Y_j . \tag{12}$$

Thus in the simplified model, the counterfactual demand for the public good depends only on the individual taste for the good and total income, and it is independent of what any planner does.[21]

It will be helpful at this point to outline the outcome of coercion - *unconstrained or* traditional social planning, as a benchmark for what is to follow. This planner chooses G and t to

[21] Thus the assumptions of Cobb-Douglas utility, defined over two goods only (with no labour - leisure choice), and exogenous income result in the same counterfactual level of G^*_j under both the '*individual-in-society*' and the '*individual-as-dictator*' approaches.

maximize the weighted sum of individual utilities $S = \sum_j z_j V_j$, $\sum_j z_j=1$, subject to the government budget restraint. (The introduction of the weights z_j will facilitate later comparison of social planning with a collective choice processes.) Using the covariance formula $\Sigma z\gamma = N(\sigma^2_{z\gamma} + \bar{z}\bar{\gamma})$, where a bar denotes the mean value of a variable, and exploiting the equality $N\bar{z} = 1$, maximization of welfare S subject (only) to the government budget constraint yields the optimal policy:

$$G^{OT} = \left(\bar{\gamma} + N\sigma^2_{z\gamma}\right)\Sigma Y \quad \text{and} \quad t^{OT} = \bar{\gamma} + N\sigma^2_{z\gamma}. \tag{13}$$

The aggregate degree of coercion associated with this standard fiscal system, is not zero. Using G^{OT} in (11) shows that the degree of coercion in traditional social planning is positive and equal to

$$K^{OT} = N(\Sigma Y)^2\left[\sigma^2_{\gamma} + (N\sigma^2_{z\gamma})^2\right]. \tag{14}$$

Intuitively, coercion in a social plan rises with the magnitude of demands for the public good, because the welfare losses from departures from preferred counterfactuals are larger then; with heterogeneity of tastes for the public good, since it is harder to satisfy a more heterogeneous community with the same restricted set of policy instruments; and with the covariance between the intensity of preferences for the public good and the weights of individuals in the social welfare function, since the social planner then attaches a higher priority to the satisfaction of those with higher (than average) preference for the public good.

4.1.1 The trade – off

Observing the coercion constraint generally requires that a non-linear relationship be maintained between aggregate coercion and public sector size. Substituting from (12) into the coercion constraint (11) and using formulas for mean and variance indicates that:

$$K = N\left[G^2 - 2\bar{\gamma}(\Sigma Y)G + \left(\sigma^2{}_{\gamma} + \bar{\gamma}^2\right)\left(\Sigma Y\right)^2 \right]. \qquad (15)$$

where implicitly, $dG/dK = 1/6(G - \bar{\gamma}\Sigma Y)$. So G increases with *K* as long as its initial size is greater than $\bar{\gamma}\Sigma Y$, the standard optimal tax value *when* $\sigma^2{}_{z\gamma} = 0$, and it decreases with *K* when *G* is less than this value. This is a complicated pattern, showing the difficulty of making comparisons of the fiscal system in a coercion-constrained fiscal system and in a traditional social plan.

To derive coercion-constrained social welfare, $S = \sum_j z_j\, Vj[G(K)]$, we substitute G implicitly defined by (15) into the indirect utility functions and aggregate across citizens. The result yields the welfare-coercion trade-off. Differentiating *S* so derived with respect to *K*, we see that [22]

$$\frac{dS}{dK} = \frac{G^{OT} - G}{2N(G - \bar{\gamma}\Sigma Y)(\Sigma Y - G)G} \quad \text{and} \quad \frac{d^2S}{dK^2} = \frac{G\left(G - G^{OT}\right)\left[\left(1 + \bar{\gamma}\right)\Sigma Y - 2G\right] - G^{OT}\left(\bar{\gamma}\Sigma Y - G\right)\left(\Sigma Y - G\right)}{4N^2G^2\left(G - \bar{\gamma}\Sigma Y\right)^3\left(\Sigma Y - G\right)^2}. \qquad (16)$$

Then using (13) and the definition of aggregate private consumption, $\Sigma_j X_j = \Sigma_j Y_j - G$, shows that

$$\frac{dS}{dK} = 0 \quad \text{and} \quad \frac{d^2S}{dK^2} = \frac{-1}{4N^2\left(\sigma^2{}_{z\gamma}\right)^2 G^{OT}\left(\Sigma X_j\right)^2} < 0.$$

We can conclude that when the standard social plan is employed, welfare reaches its unconstrained maximum, and that the trade-off between social welfare and the degree of coercion is globally concave. This concave trade-off is illustrated in Figure 4. The upward sloping part of the trade-off corresponds to what we referred to as the 'consensual society' in Figure 3.

---

[22] **Proof**: Differentiation of S(K) yields $dS/dK = \Sigma z(1-\gamma)[-1/(\Sigma Y - G)]dG/dK + \Sigma z\gamma(1/G)dG/dK$. Substituting from (15) we have $dS/dK = (\Sigma z\gamma\Sigma Y - \Sigma zG)/2N(G - \bar{\gamma}\Sigma Y)(\Sigma Y - G)G$. Recalling that $G^{OT} = \Sigma z\gamma\Sigma Y$ and using the covariance formula yields the first derivative in (17). Differentiating this with respect to *K*, we obtain the following expression for the numerator of the second derivative in (17): [numerator d^2S/dK^2] = $\left[-(dG/dK)\left(-G^3 + \Sigma Y(1+\bar{\gamma})G^2 - (\Sigma Y)^2\bar{\gamma}G\right) - (dG/dK)\left(-3G^2 + 2\Sigma Y(1+\bar{\gamma})G - (\Sigma Y)^2\bar{\gamma}\right)\left(\Sigma Y\Sigma z\gamma - G\right)\right]$. Substituting for *dG/dK* and rearranging then yields the second derivative. QED

[Figure 4 here]

4.1.2 Collective choice versus social planning

The use of coercion constraints represents a general approach of analysis that is also applicable in other settings of fiscal decision making, thus allowing for a new type of comparison among systems for reaching collective choices. To explore this important extension of the approach, we now inquire as to how democracy compares to the traditional social plan in terms of their implied trade-off between welfare and coercion. We shall consider a competitive electoral system in which policy outcomes represent a balancing of the heterogeneous economic interests of citizens, as in a probabilistic spatial voting model. This model is well–described in the literature **(**see, for example, Hettich and Winer 1999, Persson and Tabellini 2000, Tridimas and Winer 2005, Adams, Merrill and Grofman 2005, and Schofield and Sened 2006) and will be outlined quickly here. Variants of this model can be used to describe equilibria in proportional or majoritarian electoral systems, but we shall retain a more general viewpoint.

There are two expected vote-share-maximizing parties, A and B, whose policy platforms converge in the competitive political equilibrium. Voting behaviour of individual citizens differs according to their economic interests including tastes for the public good, as well as according to individual political sensitivities, or propensities to switch support between parties if one of them offers the voter a more preferred fiscal system. Parties are assumed to have the same knowledge of the stochastic distribution of the characteristics of voters.

Whether a voter votes for party A depends on two components, a policy component and a non-policy component. The policy component depends on the indirect utility of the voter (specified earlier) when party A rather than B implements its proposed policy. The non-policy component, or valence, depends on the how the voter evaluates the ideology or other personal

characteristics of the competing politicians.[23] Formally, voter j supports party A if $V_j(G_A, t_A) > V_j(G_B, t_B) + \delta + s_j$, where V is again indirect utility. The valence term $(\delta + s_j)$ has two components: δ, common to all voters and uniformly distributed on $[-1/2\psi, 1/2\psi]$, and a term s_j which is an idiosyncratic preference uniformly distributed on $[-(1/2\phi_j)+\eta_j, (1/2\phi_j)+\eta_j]$.

The expected vote share that party A maximizes by choice of a fiscal system then is

$$P_A = \frac{\psi}{\Sigma_j \phi_j} \sum_j \{\phi_j [V_j(G_A, t_A) - V_j(G_B, t_B)]\} + \frac{1}{2}. \qquad (17)$$

Here ϕ_j represents the voter's political sensitivity, that is, the effect on the probability that he will support party A of a change in his well-being (that results from a proposed change in A's platform.) Analogously, B maximizes $P_B = 1 - P_A$.

Since the parties converge in the Nash electoral equilibrium, to characterize the equilibrium without loss of generality we maximize P_A with respect to G_A and t_A subject to the budget constraint facing any successful party, with G_B and t_B constant, which requires:

$$\frac{dP_A}{dG} = \Sigma_j \phi_j \frac{dV_j}{dG} = 0.$$

Since utility is Cobb-Douglas, this implies that the political equilibrium values of G and t are

$$G^{MR} = \frac{\Sigma \phi \gamma}{\Sigma \phi} \Sigma Y \quad \text{and} \quad t^{MR} = \frac{\Sigma \phi \gamma}{\Sigma \phi},$$

where subscripts are omitted for convenience.

Denoting the relative political sensitivity of voters j by $\theta_j \equiv \phi_j / \Sigma_j \phi_j$, $\Sigma \theta_j = 1$, and using the covariance formula, equilibrium fiscal structure can then be written in a convenient form easily compared to the policy (13) chosen by the unconstrained planner:

[23] Adding the stochastic valence term, which has a continuous probability distribution, introduces continuity into the expected vote-share functions of the opposing parties and by so doing eliminates the possibility of a vote cycle. Here the valence is written as a bias in favour of party B. Equilibrium also requires concavity of these objective functions, which here is assured by the form of the utility function.

$$G^{MR} = \left(\bar{\gamma} + N\sigma^2{}_{\theta\gamma}\right)\Sigma Y \qquad \text{and} \qquad t^{MR} = \bar{\gamma} + N\sigma^2{}_{\theta\gamma}. \tag{18}$$

The corresponding degree of coercion is

$$K^{MR} = N(\Sigma Y)^2\left[\sigma^2{}_{\gamma} + (N\sigma^2{}_{\theta\gamma})^2\right]. \tag{19}$$

As with (14), we see that the degree of coercion will rise with demands for the public good (that is, with incomes); with the heterogeneity of tastes for the public good (captured by $\sigma^2{}_{\gamma}$); and with the covariance between the intensity of preferences for the public good and the political sensitivity of voters ($\sigma^2{}_{\theta\gamma}$), since voters with intense preferences are politically more influential at the expense of those who (on average) would like a smaller public sector.

A comparison of planning and democratic political competition shows that

$$K^{MR} - K^{OT} = N^3\left[(\sigma^2{}_{\theta\gamma})^2 - (\sigma^2{}_{z\gamma})^2\right](\Sigma Y)^2. \tag{20}$$

If the planner weighs all citizens equally ($\sigma^2{}_{z\gamma} = 0$), coercion in a competitive political system, when such a system is described by a spatial voting model, will always *exceed* that imposed by the social planner. The reason is that majority rule introduces fiscal discrimination according to political influence in addition to that according to individual economic preferences. And since K^{OT} corresponds to the point of maximum welfare, social welfare in the democratic solution will be lower, placing us at some point on the downward sloping part of the trade-off curve.

One should note, however, that the assumptions underlying the analysis are crucial for this result. If a planner should use distributional weights, for example, and these weights are highly correlated with tastes for the public good, it becomes possible for the competitive solution to be located on the upward-sloping portion of the corresponding trade-off, to the left of the point of maximum welfare, reached by the planner who maximizes a weighted welfare function, but who does not face a coercion constraint.

Even if we return to a planning solution where all individuals are weighted equally, we may need to exercise caution in drawing inferences about actual democratic systems from figure 4. Democratic institutions often contain built-in restrictions of a constitutional or semi-constitutional nature that limit possible coercion and that are not fully reflected in the spatial voting model. If we were to compare the planner's solution to an institutionally constrained political system, it is conceivable that such a liberal democracy would be located to the left of the maximum point on the trade-off curve in the figure.

As a whole, the analysis suggests that it may be possible to reduce coercion while raising social welfare, either by imposing additional constraints on the nature of fiscal instruments (as Simons and Buchanan and Congleton suggest), or by changing the nature of the collective choice mechanism (as suggested by Wicksell and many others since). Another possibility is that the additional constraints on state action imposed by globalization may act as effective constraints on coercion in real world situations. The trade-off analysis provides a basis for considering these questions in a new and more systematic way. But what particular proposed or actual institutional changes imply in terms of a well-defined welfare-coercion trade-off remains to be formally investigated in future research.

5. Conclusion

Although coercion is a central fact in the design and operation of the public sector, normative public economics based on the planning model has not made it an explicit element of the analysis. In this paper, we formally introduce coercion into normative analysis by adding constraints that limit allowable coercion caused by tax and expenditure programs. We focus on situations that arise when citizens experience a mismatch between what they receive in public

goods and services and what they pay in taxes. The paper demonstrates that it is possible to conduct formal analysis of the structure of public policy taking coercion into account even without knowing the optimal degree of coercion. In particular, one can delineate the welfare-coercion trade-off and ask what policies are consistent with attainment of the frontier, and where particular institutions lead in relation to coercion-unconstrained social planning.

To make the concept of coercion operational, a counterfactual specifying what individuals regard as appropriate treatment by the public sector is required. We have employed a counterfactual that assumes that the individual accepts some coercion by society, along with a socially determined tax-price. One may then specify coercion constraints either in terms of individual or aggregate utility, or by using a convenient approximation that relies on a reference level of government expenditure. The aggregate definitions are analogous to the use of the Hicks-Kaldor criterion and impose a less severe constraint on decision making than those having an individual basis.

Coercion constraints have important and complex effects on a social plan. Using both aggregate (Hicks-Kaldor-like) and individual coercion constraints, we work out these effects for a fiscal system that uses an optimal linear income tax to provide a public good and, in the Appendix (using an aggregate definition of coercion), for the implementation of an optimal consumption tax in the presence of an optimal linear income tax. These cases were chosen because they permit straightforward comparisons with standard optimal tax results, including the size of government, the marginal cost of funds (MCF), and the pattern of commodity taxes in the presence of an optimal income tax.

A novel aspect of the analysis relates to the trade-off between social welfare as traditionally defined and coercion. Using a Cobb-Douglas formulation, we derive a trade-off

function, as well as the degree of coercion implied by unconstrained social planning. The analysis allows us to examine how to achieve the highest level of traditionally defined welfare for a given degree of coercion or, in other words, how to be coercion-efficient. The trade-off between narrowly defined welfare and aggregate coercion raise the possibility that collective choice in a democracy will tend to lead us to a point on the downward sloping part of the trade-off, opening up the possibility that coercion may be reduced and social welfare increased by appropriate institutional reform.

Extensions of the analysis are possible is several directions. One could, for example, explicitly account for the interaction of incentive compatibility and coercion constraints. Such interaction would occur in situations where the coercion of individuals in different income groups is relevant to decision making by those who may find it advantageous to mimic the behavior of others. In addition, coercion will have relevance for the structure of the fiscal system. Although we have considered coercion when only an income tax is employed, or when a commodity tax is used along with income taxation, the analysis could be extended to situations where a full tax mix exists.[24] More generally, the relationship between complexity of tax structures and the coercion-welfare trade-off also deserves investigation. One suspects that more complex fiscal systems, which are also administratively more costly, may involve less coercion for a given level of welfare. [25]

The trade-off analysis can also be used to investigate how coercion can be reduced at given levels of social welfare through institutional means. Work on the scope of the public sector suggests that the boundary between private and public sectors matters in this regard, and that the

[24] See, for example, Boadway and Marchand (1995**)** on incentive compatibility and public expenditure, and Hettich and Winer (1988, 1999) on the formation of tax structure. Compared to the existing literature, a new element in the work on tax structure will be preferences for public goods, because coercion depends in part on such preferences.

[25] Yitzhaki (1979) and Hettich and Winer (1999) have dealt with tax complexity, but not in a framework that explicitly acknowledges coercion.

welfare-coercion frontier may be shifted favorably by removing certain types of economic activity from the public sphere. The trade-off function could be used to formalize this argument.

Public goods coercion also has relevance for the discussion of federalism. Following Tiebout (1956), the literature on optimal assignment in federations has been concerned with balancing the welfare gains from decentralization with the loss of efficiency from fiscal externalities that arise under decentralized decision making. One may expect decentralization, or the principle of subsidiarity, to reduce coercion, but this connection has not yet been formally analyzed in the optimal assignment literature using a welfare - coercion trade-off such as the one analyzed here.[26]

Finally, the welfare-coercion frontier also allows us to extend the analysis of collective choice in an important way. The concept provides a new basis for comparing political equilibria under alternative institutional arrangements or voting rules, and for the ranking of such equilibria with respect to the implied trade-off between welfare and coercion.

[26] For reviews of the literature, see Wildasin (2006) and Wilson (1999). Pennock (1959) analyzed the relationship between majority rule and federalism, arguing that decentralization increases the total number of citizens in a majority coalition. But while this suggests that decentralization reduces coercion, he did not measure coercion formally nor integrate efficiency into his argument.

Appendix

1. Linear income taxation with individual coercion constraints

When the planner is constrained by how much he or she can coerce each *individual* taxpayer separately (as in case 1 of Table 1), the Lagrangean for the planning problem becomes

$$\mathcal{L} = \sum_j V_j + \mu\,[t\sum_j W_j H_j - Na - P_G G] + \sum_j \kappa_j\,[\,K_j - (V^*_j - V_j)]\,. \qquad \text{(A1)}$$

Although the situation is considerably more complex than before, the corresponding first order conditions are generalizations of equations (7) and are not stated here. Working as before, the condition for the optimal coercion-constrained size of government (analogous to condition (8)) becomes [27]

$$\left(\sum_j m_j\right)\left[(1+\delta)(1+\kappa)+\kappa\phi\right] = \frac{\mu}{\lambda}\left(P_G - t\sum_j W_j \frac{\partial H_j}{\partial G}\right) \qquad \text{(A2)}$$

where in addition to previous definitions, $s_{\kappa\lambda} = \sigma^2_{\kappa\lambda}/\lambda\kappa$ is the covariance between coercion and the marginal utility of income; $s_{\kappa m} = \sigma^2_{\kappa m}/\kappa m$ is the covariance between coercion and the marginal rate of substitution; $s_{\kappa\lambda m} = \sigma^2_{\kappa\lambda m}/\kappa\lambda m$ is the covariance between coercion, the marginal utility of income and the marginal rate of substitution; and finally, where

$$\phi \equiv\ s_{\kappa\lambda} + s_{\kappa m} + s_{\kappa\lambda m}\,.$$

The right-hand-side of equation (A2) is already familiar. It is the product of the marginal valuation of government revenue times the net marginal rate of transformation of the public good. The left-hand-side of (A2) again shows the marginal benefit from the public good. But now it is the product of the sum of marginal rates of substitution multiplied by the adjustment for the combined effect of the distributional characteristics of the public good *and* the effects of coercion. In the present case of individual coercion constraints, the adjustment for coercion contains two new elements relative to standard social planning, (i) the average effect of coercion $(1+\delta)(1+\kappa)$, a term that also appears in the previous case of aggregate coercion, and (ii) the term $\kappa\phi$, which corrects the aggregate term for the 'distributional characteristics of coercion'.

We use the term 'the distributional characteristics of coercion' advisedly here, for want of a better one. Since concern with coercion arises out of concern with individual rights or, alternatively, with the degree of social solidarity individuals have with the objectives of the planner, it is not clear that we ought to think about it the same way that we do redistribution in the traditional planning model.

---

[27] To derive (A2), note that the analogue to first order condition (7.3) when coercion constraints apply to individuals is: $\sum_j (1+\kappa_j)\lambda_j m_j = \mu\,[P - t\sum_j W_j(\partial H_j/\partial G)]$. The left-hand-side of this can be written as $\sum_j(1+\kappa_j)\lambda_j m_j = \sum_j \lambda_j m_j + \sum_j \kappa_j\lambda_j m_j$. Recall that κ, λ, and m are the means of κ_j, λ_j, and m_j respectively: $cov(\kappa_j\lambda_j) = (1/N)\sum_j(\lambda_j - \lambda)(m_j - m)$ and $cov(\kappa_j\lambda_j m_j) = (1/N)\sum_j(\kappa_j - \kappa)(\lambda_j - \lambda)(m_j - m)$. Manipulating the covariances and using the new first order condition yields intermediate steps: $\sum_j\lambda_j m_j = N\sigma^2_{\lambda m} + \lambda\sum_j m_j$ and $\sum_j\kappa_j\lambda_j m_j = [\kappa\lambda\sum_j m_j + N(\kappa\sigma^2_{\lambda m} + \lambda\sigma^2_{\kappa m} + m\sigma^2_{\kappa\lambda} + \sigma^2_{\kappa\lambda m})]$.

Now the benefit from public provision increases in the following cases, assuming κ is positive: (i) If the rich (low λ_j) view the payoff from solidarity with the planner less favourably (low κ), so that $\sigma^2_{\kappa\lambda} > 0$ and $s_{\kappa\lambda} > 0$; (ii) If those who value public goods less (low m_j) 'have less social solidarity' (low κ), then $\sigma^2_{\kappa m} > 0$ and $s_{\kappa m} > 0$; and (iii) If the rich (low λ_j) also value public goods less (low m_j) , so that $\sigma^2_{\kappa\lambda m} > 0$ and $s_{\kappa\lambda m} > 0$, because the previous two effects are compounded.

If all these conditions apply, ϕ is positive. Then on comparing (8) and (A2) one can also say that the Hicks-Kaldor-like solution for a coercion-constrained optimum (8) will involve *less* spending and a lower tax rate than when coercion is defined on an individual basis. However, either of these comparisons could in principle go the other way, and it will be interesting to determine in practice what situation is likely to apply.

In order to derive the optimal income tax rate under individual coercion constraints, we require additional covariances (normalized again by the means of the indicated variables): $\sigma^2_{\kappa\lambda Y}$ = the covariance of κ_j, λ_j and Y_j; $\sigma^2_{\kappa\psi q}$ =*the* covariance of κ_j, ψ_j and q_j; $\sigma^2_{\kappa Y}$ = the covariance of κ_j and Y_j; $\sigma^2_{\psi q}$ = the covariance of ψ_j and κ_j; $\sigma^2_{\kappa q}$ = the covariance of κ_j; and q. Also, let κ, λ, ψ and q be the mean values of κ_j, λ_j, ψ_j and q_j respectively. Then using these definitions and the Slutsky equation, and working as before, we obtain the formula for the coercion-constrained optimal income tax rate [28]:

$$t = \frac{(1+\kappa)\sigma^2_{\lambda Y} + \kappa\sigma^2_{\psi q} + \lambda\sigma^2_{\kappa Y} + \psi\sigma^2_{\kappa q} + \sigma^2_{\kappa\lambda Y} + \sigma^2_{\kappa\psi q}}{\mu\left(\overline{WS} - \sigma^2_{Ya}\right)}. \qquad \text{(A3)}$$

This optimal income tax rate depends as usual on the income distribution effect of taxation, captured by $\sigma^2_{\lambda Y}$, and the efficiency effect of taxation on labour (shown again by the denominator). In common with the case of aggregate coercion, it also depends on the relationship between the marginal utility of the tax share and the marginal tax share $\sigma^2_{\psi q}$. In addition, the optimal tax rate depends on the distributional effects of coercion, as the remaining four covariance terms make clear. It should be noted that little is presently known about the sign or size of the covariances involved.

2. The optimal structure of indirect taxation in the presence of a coercion constraint

We show how adding a concern with coercion alters the structure of an optimal system consisting of both income and consumption taxation. Again to preserve comparability with well known results in the literature, we utilize the analysis of Atkinson and Stiglitz (1980). In this case, the service provided by government is a lump–sum transfer 'a', provided equally to all citizens, and coercion is defined in terms of the balance between this uniform transfer and what each person pays in total taxes.

---

[28] To proceed, one multiplies the analogue to (7.1) for individual constraints by (1/N) and that for (7.2) by $(\sum_j W_j H_j / N^2)$ and subtracts the latter from the former. The left-hand-side of the result involves the distributions of three variables, the individual coercion constraint κ_j, the marginal utility of income λ_j, and income Y_j. Similarly, the right-hand-side features the individual coercion constraint κ_j, the marginal utility of the tax share $\psi_j = -\lambda^*_j PG^*_j$, and the marginal tax share, $q_j = [(\partial\tau_j/\partial t) + (\partial\tau_j/\partial a)\ \bar{Y}]$, as well as the effect of income taxation on labour supply. Applying the definitions of covariances in the text then yields (A3).

We are interested in how the introduction of an aggregate coercion constraint into the planner's problem alters the optimal structure of this fiscal system. Intuition leads one to suspect that a concern with coercion will alter the direct-indirect tax mix and the structure of each tax. The reason is that since taxation of each individual depends on his or her consumption pattern, so does coercion, and that optimizing social welfare in the face of a coercion constraint will then require that taxation takes this pattern of consumption into account.

So, let the fiscal system consist of a linear income tax (as analyzed in the text) with a uniform proportional rate t and a lump sum transfer a, ***and*** a consumption tax that consists of a unique rate of tax t_i on each commodity. There are J individuals, and the utility of person j depends on the quantity X_i, $i = 1,\ldots,n$ of good i and leisure L: $U^j = U^j(X^j_i, L_j)$, $j=1,\ldots,N$. Producer prices are constant and normalized to unity. Consumer prices are inclusive of the commodity tax and can be expressed as $q_i = 1+ t_i$. Each individual maximizes utility subject to the budget constraint $\Sigma_i q_i X_i^j = W^j(1-L^j) + a$ where W^j is the exogenous wage rate.

The tax schedule for each person takes the form $T^j = t_i X_i^j - a$, and the budget constraint of the government is $\Sigma_j \Sigma_i \; t_i X_i^j = Ja$. Here the uniform rate of the income tax t is subsumed into the individual rates on each commodity. This can be done to simplify the way in which tax structure is represented, because a uniform proportional tax on total income is equivalent to a uniform tax on total consumption. Citizen - taxpayers are assumed to pay too much tax in comparison to what they think they should receive in transfers. Without loss of generality, in defining coercion we focus here exclusively on the taxpayers who think they pay too much and whose number is assumed to be $M < J$. When the resulting aggregate coercion constraint binds, taxes must be set so that the overpayment made does not exceed a given sum K.

$$\sum_i t_i X_i^j - a = K \tag{A4}$$

The coercion-constrained social planner is assumed to choose the tax rate t_i the lump sum transfer a to maximize a social welfare function defined over individual indirect utilities $\Psi = \Psi(V^1, \ldots, V^N)$ subject to the budget constraint and the coercion constraint. The Lagrangean is

$$\Lambda = \Psi\left(U^j\right) + \mu\left[\sum_j \; \sum_i t_i X_i^j - Ja\right] + \kappa \sum_m \; \left[K - \left(\sum_i t_i X_i^m - a\right)\right] \tag{A5}$$

Differentiating with respect to the tax on good g, t_g , and the lump-sum payment, a, we obtain the first order conditions

$$\sum_j \frac{\partial \Psi}{\partial V^j} \frac{\partial V^j}{\partial t_g} + \mu \left[\sum_j X_g^j + \sum_j \; \sum_i t_i \frac{\partial X_g^j}{\partial t_g} \right] - \kappa \left[\sum_m X_g^m + \sum_m \; \sum_i t_i \frac{\partial X_g^m}{\partial t_g} \right] = 0 \tag{A6.a}$$

$$\sum_j \frac{\partial \Psi}{\partial V^j} \frac{\partial V^j}{\partial a} + \mu \left[\sum_j \; \sum_i t_i \frac{\partial X_g^j}{\partial a} - J \right] - \kappa \left[\sum_m \; \sum_i t_i \frac{\partial X_g^m}{\partial a} - M \right] = 0 \cdot \tag{A6.b}$$

Denoting the marginal utility of income of the jth consumer by λ^j and using Roy's identity, $\partial V^j/\partial t_g = -\lambda^j X^j_g$ and $\partial V^j/\partial a = \lambda^j$. Let $\beta^j \equiv (\partial \Psi/\partial V^j)\lambda^j$ denote the social marginal utility of income. We may then write

$$(\partial \Psi/\partial V^j)(\partial V^j/\partial t_g) = -\beta^j X^j{}_g \quad \text{(A7.a)}$$
$$(\partial \Psi/\partial V^j)(\partial V^j/\partial a) = \beta^j \quad \text{(A7.b)}$$

Using the Slutsky equation, $\partial X_i^{\,j} / \partial t_g = S_{ig}^j - (\partial X_i^{\,j} / \partial a) X_g^{\,j}$, where S_{ig}^j denotes the compensated effect of a change in the tax on good *g* to the demand for good *i*, and $\partial X_i^{\,j} / \partial a$ denotes the income effect, and substituting from (A7), the first order conditions can then be rewritten as:

$$-\mu \sum_j \left[\frac{\beta^j}{\mu} + \sum_i t_i \frac{\partial X_g^{\,j}}{\partial a} \right] X_g^{\,j} + \mu J \overline{X_g} + \mu \sum_j \sum_i t_i S_{ig}^j - \kappa \sum_j \left[1 - \sum_i t_i \frac{\partial X_g^{\,m}}{\partial a} \right] X_g^{\,m} - \kappa \sum_m \sum_i t_i S_{ig}^m = 0 \quad \text{(A8.a)}$$

$$\mu \sum_j \left[\frac{\beta^j}{\mu} + \sum_i t_i \frac{\partial X_g^{\,j}}{\partial a} \right] - \mu J - \kappa \sum_m \sum_i t_i \frac{\partial X_g^{\,m}}{\partial a} + \kappa M = 0 . \quad \text{(A8.b)}$$

Let

$$b^j \equiv \left(\beta^j / \mu \right) + \Sigma_i t_i \left(\partial X_g^{\,j} / \partial a \right) \quad \text{(A9)}$$

denote the *net social marginal valuation of income* measured in terms of government revenue. It is net in the sense that it measures the benefit from transferring \$1 to household *j* correcting for the tax paid upon receiving the extra \$1. Similarly, let

$$c^j \equiv 1 - \Sigma_i t_i \left(\partial X_g^{\,m} / \partial a \right) \quad \text{(A10)}$$

denote *the net marginal valuation of coercion* for person *j*. The latter reflects the benefit to individual m of relaxing coercion by one monetary unit net of the tax paid.

Using the above two definitions in (A8) and rearranging gives the following restatement of the first order conditions:

$$\sum_j \sum_i t_i S_{ig}^j - \frac{\kappa}{\mu} \sum_m \sum_i t_i S_{ig}^m = \sum_j b^j X^j - J \overline{X_g} + \frac{\kappa}{\mu} \sum_m c^m X_g^{\,m} \quad \text{(A11.a)}$$

$$\sum_j b^j = J - \frac{\kappa}{\mu} \left(M - \sum_m c^m \right) \quad \text{(A11.b)}$$

Following Atkinson and Stiglitz (1980), we define the normalized covariance between the net social marginal valuation of income (b^j) and the consumption of the gth commodity ($X^j{}_g$), where a bar indicates the mean value of a variable, as

$$\phi_g \equiv \frac{1}{J} \sum_j \frac{b^j}{\bar{b}} \frac{X_g^{\,j}}{\overline{X_g}} - 1 \quad \text{(A12)}$$

This covariance shows *the distributional characteristic of good g* and depends on the pattern of for g and on the behaviour of the net social marginal valuation of income. Similarly, we also define the normalized covariance between $X^m{}_g$ and c^m as

$$\omega_g \equiv \frac{1}{M}\sum\nolimits_m \frac{c^m}{\bar{c}}\frac{X_g^m}{\overline{X_g^m}} - 1 \qquad \text{(A13)}$$

In turn, the above covariance shows *the coercive characteristic of good g* and depends on the pattern of demand for good g by the M consumers who are coerced and c^m.

Substituting from (A12) and (A13) into (A11.a) and (A11.b), denoting the mean of the net social marginal valuation of income measured in terms of government revenue by $\bar{b}$, and manipulating:

$$\frac{1}{J\overline{X_g}}\left\{\sum\nolimits_j \sum\nolimits_i t_i S_{ig}^j - \frac{\kappa}{\mu}\sum\nolimits_m \sum\nolimits_i t_i S_{ig}^m\right\} = \bar{b}\phi_g - \left(1-\bar{b}\right) + \frac{\kappa}{\mu}\frac{M}{J}\frac{\overline{X^m{}_g}}{\overline{X_g}}\bar{c}\left(\omega_g - 1\right) \qquad \text{(A14.a)}$$

$$\bar{b} = 1 - \frac{\kappa}{\mu}\frac{M}{J}\bar{c} \qquad \text{(A14.b)}$$

Each side of equation (A14) contains two components. The first component of the left side shows the proportional reduction in the consumption of the *gth* good by all *J* taxpayers resulting from taxing along the compensated demand curves of all the i=1,…,n goods, $\sum_j \sum_i t_i S_{ig}^j$. The second component, $\frac{\kappa}{\mu}\sum_m \sum_i t_i S_{ig}^m$, shows the proportional reduction in the consumption of g from the tax on the M taxpayers who are coerced, again along the compensated demand curves. In the standard treatment of the structure of taxation only the former term appears (see Atkinson & Stiglitz 1980, p. 431). Thus, the left side of (A14.a) represents the coercion-adjusted proportional reduction in the compensated demand of the gth good from taxing g.

Turning to the right side of (A14.a), the term $\bar{b}\phi_g - \left(1-\bar{b}\right)$ shows the mean net social marginal valuation of income of consumers, measured in terms of government revenue, adjusted for the differences in their incomes. The second term, $\frac{\kappa}{\mu}\frac{M}{J}\frac{\overline{X^m{}_g}}{\overline{X_g}}\bar{c}\left(\omega_g - 1\right)$, shows the mean effect of the coercion constraint on the social valuation of income, adjusted for the differences in the income of different individuals. Again, only the former features in the standard treatment of the structure of indirect taxation problem. Thus, the right hand side represents the coercion-adjusted net social marginal valuation of income.

The equality in (A14.a) requires that for optimality, the reduction in the compensated consumption of the gth good due to commodity taxation, after accounting for the effect of coercion, must be equal to the change in the net social marginal valuation of income also accounting for the effect of coercion.

Equation (A14.b) shows that, unlike the standard case of optimal income tax, the mean net marginal social valuation of \$1 of government revenue is less than 1 by the factor $\kappa M\bar{c}/\mu J$, which measures the *mean effect of the coercion* on taxpayers.

For a better understanding of these findings we compare them with the standard results of the structure of indirect taxes presented by Atkinson and Stiglitz (1980). From equations (A14.a), the standard formulas of Atkinson and Stiglitz without coercion are obtained upon assuming $\kappa = 0$ (again see p.431)

$$\frac{1}{J\overline{X}_g}\left\{\sum_j \sum_i t_i S_{ig}^j\right\} = \bar{b}\phi_g - \left(1-\bar{b}\right) \qquad \text{(A15.a)}$$

$$\bar{b} = 1 \qquad \text{(A15.b)}$$

If the government is only interested in efficiency, that is, it does not pursue any distributional objectives, then $\phi_g = 0$, for all goods g and the right side of (A15.a) reduces to $-(1-\bar{b})$, which is the same for all commodities. Therefore, for optimality the tax should be set (following Ramsey 1927) so that there is a uniform percentage reduction in compensated demands for all commodities.

On the other hand, when the lump-sum component of the income tax - transfer system, a, is chosen efficiently by the planner, (A15.b) is operative, which, in turn implies that the right side of (A15.a) is equal to ϕ_g. In this case, the indirect tax on each good g should vary according to its distributional characteristic ϕ_g, and indirect taxes are now used for pursuing the distribution objective of the government, while efficiency is served by the optimal lump-sum component a.

These results are modified substantially in the presence of the coercion constraint. If the government does not pursue any equity objectives, then $\phi_g = \omega_m = 0$, for all goods g. The right side of (A14.a) is then equal to $-\left(1-\bar{b}\right) - \frac{\kappa}{\mu}\frac{M}{J}\frac{\overline{X^m}_g}{\overline{X}_g}\bar{c}$, which in general depends on good g. It then follows that when only efficiency matters, but coercion constraints are taken into account, and contrary to the standard case, indirect commodity taxes should **not** be set at uniform levels, but vary with the characteristics of the good on which the tax is levied. The intuition is straightforward – coercion depends in part of how much commodity tax one pays, and this depends on the pattern of consumption.

Moreover, when the lump-sum component of the tax, a, is optimally chosen, (A14.b) becomes operative, and the right side of (A15.a) becomes equal to $\phi_g - \frac{\kappa}{\mu}\frac{M}{J}\bar{c}\left(1+\phi_g+\frac{\overline{X^m}_g}{\overline{X}_g}\left(1-\omega_g\right)\right)$. This of course depends on the demand characteristics of the taxed good g, the equity objective and the coercion constraint, so that now indirect taxes should differ across different commodities ***and*** should also be used to pursue equity objectives.

It seems reasonable to us to suggest that coercion plays a role in determining the nature of the fiscal system employed by governments that are responsive to the economic interests of citizens. The complicated (and non-uniform) pattern of consumption taxes actually observed in most advanced democracies is this likely to be partly shaped by what citizens see as the degree of coercion they are subjected to in the course of carrying out the state's fiscal affairs.

3. Derivation of $d\kappa/dK$ in Section 4, showing that in the case of linear income taxation and an aggregate coercion constraint, the sign of this derivative is ambiguous.

Maximization of the welfare function subject to the government budget and the coercion constraint generate a system of 5 equations with five unknowns: the three first order conditions (7.1), (7.2) and (7.3), the government budget restraint (5), and the aggregate coercion constraint $\sum_j (V^*_j - V_j) = K$.

Here there are five unknowns: the three fiscal parameters, *t, a* and *G*, and the two Lagrange multipliers, μ (for the budget constraint) and κ (for the coercion constraint). Solving the above system gives us the formulas for, *t*, *a* and *G* that we discuss in the text, and also the solutions for μ and κ.

As noted in the text, the equilibrium values of the endogenous variables are a function of the distribution of individual tastes for work, leisure and consumption, captured by the parameters of the utility function and denoted by $\boldsymbol{\Gamma}$; the characteristics of the distribution of earning abilities captured by the wage rates and denoted by $\boldsymbol{W}$; the price of the public good P; and the degree of (aggregate) coercion *K*. We may write the system of reduced-form equations as:

$$t = t(\boldsymbol{\Gamma}, \boldsymbol{W}, P, K); \quad a = a(\boldsymbol{\Gamma}, \boldsymbol{W}, P, K); \; G = G(\boldsymbol{\Gamma}, \boldsymbol{W}, P, K); \quad \kappa = \kappa(\boldsymbol{\Gamma}, \boldsymbol{W}, P, K); \quad \mu = \mu(\boldsymbol{\Gamma}, \boldsymbol{W}, P, K). \tag{A16}$$

Using the assumptions of the linear tax model in section four, totally differentiating the system of equations (A16), using subscripts to denote derivatives and rearranging gives the following:

$$\begin{bmatrix} A_{tt} & A_{ta} & A_{tG} & -\alpha_{t\mu} & \alpha_{t\kappa} \\ B_{at} & B_{aa} & B_{aG} & -\beta_{a\mu} & \beta_{a\kappa} \\ C_{Gt} & C_{at} & C_{GG} & -\gamma_{G\mu} & \gamma_{G\kappa} \\ \alpha_{t\mu} & -\beta_{a\mu} & -\gamma_{G\mu} & 0 & 0 \\ \alpha_{t\kappa} & -\beta_{a\kappa} & -\gamma_{G\kappa} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} dt \\ da \\ dG \\ d\mu \\ d\kappa \end{bmatrix} = \begin{bmatrix} 0 \\ \mu dN \\ \mu dP + adN \\ GdP \\ dK \end{bmatrix} \tag{A17}$$

where:

$A_{tt} \equiv (1+\kappa)\Sigma(\lambda_t Y + \lambda Y_t) - \mu(2\Sigma Y_t + t\Sigma Y_{tt}) - \kappa\Sigma\lambda^* PG^* \tau_{tt}$; $A_{ta} \equiv (1+\kappa)\Sigma(\lambda_a Y + \lambda Y_a) - \mu(\Sigma Y_a + t\Sigma Y_{ta}) - \kappa\Sigma\lambda^* PG^* \tau_{ta}$; $A_{tG} \equiv (1+\kappa)\Sigma(\lambda_G Y + \lambda Y_G) - \mu(\Sigma Y_G + t\Sigma Y_{Gt})$; $\alpha_{t\mu} \equiv \Sigma Y + t\,\Sigma Y_t$; $\alpha_{t\kappa} \equiv \Sigma\lambda Y - \Sigma\lambda^* PG^* \tau_t$;

$B_{at} \equiv (1+\kappa)\Sigma\lambda_t + \mu(\Sigma Y_a + t\Sigma Y_{at}) + \kappa\Sigma\lambda^* PG^* \tau_{at}$; $B_{aa} \equiv (1+\kappa)\Sigma\lambda_a + \mu t\Sigma Y_{aa} + \kappa\Sigma\lambda^* PG^* \tau_{aa}$; $B_{aG} \equiv (1+\kappa)\Sigma\lambda_G + \mu\Sigma Y_{aG}$; $\beta_{a\mu} \equiv N - t\Sigma Y_{at}$; $\beta_{a\kappa} \equiv \Sigma\lambda + \Sigma\lambda^* PG^* \tau_a$; *and*

$C_{Gt} \equiv (1+\kappa)\Sigma U_G + \mu(\Sigma Y_G + t\Sigma Y_{Gt})$; $C_{Ga} \equiv (1+\kappa)\Sigma U_{Ga} + \mu\Sigma Y_{Ga}$; $C_{GG} \equiv (1+\kappa)\Sigma U_G + \mu t\Sigma Y_{GG}$; $\gamma_{G\mu} \equiv P - t\Sigma Y_G$; $\gamma_{G\kappa} \equiv \Sigma U_G$.

By Cramer's rule,

$$\frac{d\kappa}{dK} = \frac{|D|}{|\Omega|} \tag{A18}$$

where

$|D| = \begin{vmatrix} A_{tt} & A_{ta} & A_{tG} & -\alpha_{t\mu} \\ B_{at} & B_{aa} & B_{aG} & -\beta_{a\mu} \\ C_{Gt} & C_{Ga} & C_{GG} & -\gamma_{G\mu} \\ \alpha_{t\mu} & -\beta_{a\mu} & -\gamma_{G\mu} & 0 \end{vmatrix}$ and $|\Omega|$ is the determinant of the matrix of coefficients in (A17).

This derivative cannot be signed unambiguously even with the assumptions of the simple linear income tax model, because none of the individual terms in the determinants of D and Ω can be signed without making further assumptions.

References

Acemoglu, Daron, Michael Golosov and Aleh Tsyvinski (2008). "Dynamic Mirrlees Taxation under Political Economy Constraints". Working paper 08-08, February 15. Cambridge Mass: MIT

Adams, J. F., Merrill III, S., Grofman, B. (2005). *A Unified Theory of Party Competition. A Cross-National Analysis Integrating Spatial and Behavioural Factors*. Cambridge University Press.

Alesina, Alberto and Enrico Spolaore (2003). *The Size of Nations.* MIT Press.

Alm, James, Gary H. McClelland and William D. Schulze (1992). "Why Do People Pay Taxes?"**.** *Journal of Public Economics* 48(1), 21-38

Anderson, Scott (2006). "Coercion. Stanford Encyclopedia of Philosophy, February. At http: //plato.stanford.edu/entries/coercion

"Atkinson, Anthony B. and Nicholas H. Stern (1974). "Pigou, Taxation and Public Goods". *Review of Economic Studies* 41, 119 - 128.

Atkinson, Anthony B. and Joseph E. Stiglitz (1980). *Lectures on Public Economics*. New York: McGraw-Hill.

Atkinson, Anthony and Joseph Stiglitz (1976). "The Design of Tax Structure: Direct versus Indirect Taxation". *Journal of Public Economics* 6, 55-75.

Becker, Gary (1983). "A Theory of Competition Among Pressure Groups for Political Influence". *Quarterly Journal of Economics* 98, 371-400.

Boadway, Robin (2002). "The Role of the Public Choice Considerations in Normative Public Economics". In Stanley L. Winer and Hirofumi Shibata (eds.) *Political Economy and Public Finance.* Edward Elgar, 47-68.

Boadway, Robin and Maurice Marchand (1995). "The Use of Public Expenditures for Redistributive Purposes". *Oxford Economic Papers*, New Series, 47 (1), 45-59.

Breton, Albert (1974). *The Economic Theory of Representative Government*. Chicago: Aldine.

Breton, Albert. (1996). *Competitive Governments: An Economic Theory of Politics and Public Finance*. Cambridge University Press.

Buchanan, James M. (1964). "Fiscal Institutions and Efficiency in Collective Outlay". A*merican Economic Review, Papers and Proceedings*, May, 227 - 235.

Buchanan, James M. (1968). *The Demand and Supply of Public Goods*. Rand McNally

Buchanan, James M. and Roger Congleton (1998). *Politics by Principle, Not Interest*. Cambridge University Press.

Buchanan, James M. and Gordon Tullock (1962). *The Calculus of Consent. Logical Foundations of Constitutional Democracy*. University of Michigan Press.

Clower, Robert (1967). "A Reconsideration of the Microfoundations of Monetary Theory", *Western Economic Journal* 6, 1 - 8.

Dalton, Thomas (1977). "Citizen Ignorance and Political Activity". *Public Choice, 85 - 99.*

Escarrez, D. R. (1967) "Wicksell and Lindahl: Theories of Public Expenditure and Tax Justice Reconsidered". *National Tax Journal* 20, 137-148.

Gordon, Scott (1999). *Controlling the State: Constitutionalism from Ancient Athens to Today***.** Cambridge, Mass.: Harvard University Press.

Hart. H. L.A. (1961). *The Concept of Law*. Oxford: Oxford University press

Hettich, Walter and Stanley L. Winer (1985). "Blueprints and Pathways: The Shifting Foundations of Tax Reform," *National Tax Journal*, 38(4): 423-45.

Hettich, Walter and Stanley L. Winer (1988). "Economic and Political Foundations of Tax Structure," *American Economic Review* 78(4), 1988, 701-712.

Hettich, Walter and Stanley L. Winer (1999). *Democratic Choice and Taxation: A Theoretical and Empirical Investigation*. Cambridge University Press.

Hirschman, Albert O. (1970) *Exit, Voice and Loyalty: Responses to Declines in Firms, Organizations and States.* Cambridge University Press**.**

Kaplow, Louis (2008). "Pareto Principle and Competing Principles" in (S. Durlauf and L. Blume eds. *The New Palgrave Dictionary of Economics.* Macmillan.

Levy, Margaret (1997). *Consent, Dissent, and Patriotism*. Cambridge University Press.

Leiser, B.M. (2008). "On Coercion". in D. Reidy and W. Riker eds. (2008). *Coercion and the State*. Springer, 31 – 43.

Lindahl, Eric (1919). "Just Taxation, A Positive Solution". in Richard Musgrave and Alan Peacock (1958). *Classics in the Theory of Public Finance*. Macmillan, 168-176.

Mirrlees, James A. (1971). "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175-208.

Mueller, Dennis (2003). *Public Choice III*. Cambridge University Press.
Patinkin, Don (1965). *Money, Interest and Prices: An Integration of Monetary and Value Theory*. 2nd Edition. Harper and Row.

Pennock, Roland (1959). "Federal and Unitary Government - Disharmony and Frustration". *Behavioral Science* 4, 147- 157.

Perroni, Carlo and Kimberley Scharf (2003). "Viable Tax Constitutions". Unpublished, CEPR Working Paper DP 4210, University of Warwick.

Persson, Torsten and Guido, Tabellini (2000). *Political Economics. Explaining economic policy*. Cambridge MA:, MIT Press

Ramsey, F. P. (1927)."A Contribution to the Theory of Taxation". *Economic Journal* 37, 47-61.

Reidy, David A. and Walter J. Riker eds. (2008). *Coercion and the State*. Springer.

Riker, William H. (1982). Liberalism Against Populism: A Confrontation Between the Theory of Democracy and the Theory of Social Choice**.** W.H. Freeman & Company

Sandmo, Agnar (1998). "Redistribution and the Marginal Cost of Public Funds" *Journal of Public Economics* 70, 365-382.

Simons, Henry C. (1938). *Personal Income Taxation: The Definition of Income as a Problem of Fiscal Policy*. Chicago: University of Chicago Press.

Schofield, N. and Itai Sened. (2006). *Multiparty Democracy: Parties, Elections and Legislative Politics in Multiparty Systems*. Cambridge University Press.

Skaperdas, Stergios (1992). "Cooperation, Conflict, and Power in the Absence of Property Rights". *American Economic Review* 82, 720-739.

Tiebout, Charles (1956). "A Pure Theory of Local expenditures". *Journal of Political Economy* 64, 416 - 424.

Tridimas, George and Stanley L. Wine (2005). "The political economy of government size". *European Journal of Political Economy* 21, 643–666

Usher, Dan (1993). *The Welfare Economics of Markets, Voting and Predation*. University of Michigan Press.

Wicksell, Knut (1896). "A New Principle of Just Taxation". in R. Musgrave and A. Peacock (1958). *Classics in the Theory of Public Finance*. Macmillan, 72-118.

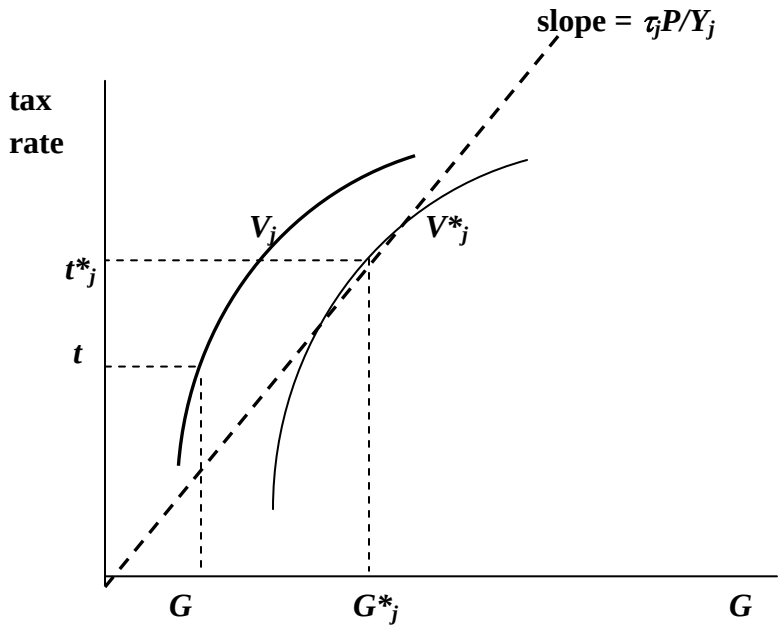
Wildasin, David (2006). "Fiscal Competition", in Barry R. Weingast and Donald A. Wittman (eds.). *The Oxford Handbook of Political Economy*, 502 - 520.

Wilson, John (1999). "Theories of Tax Competition". *National Tax Journal* 52, 269 - 304.

Wiseman, Jack (1989). *Cost, Choice and Political Economy*. Edward Elgar.

Yitzhaki, Shlomo (1979). "A Note on Optimum Taxation and Administrative Costs", *American Economic Review* 69, 475-80.

Figure 1
The Individual-in-Society Counterfactual



Legend:

t actual tax rate paid.

$t^*_j = G^*_j(\tau_j P/Y_j)$ **:** the implied income tax rate tax at which the *individual-in-society* is assumed to be able to quantity-adjust the level of the public good, given his tax-share τ_j, the price of the public good P and income Yj.

G actual level of the public good.

G^*_j level of the public good that the individual would like the community to provide at his given tax price

V^*_j maximum desired utility at the individual's given tax price if that person could quantity- adjust the level of the public good.

$V^*_j - V$ coercion when the *individual-in-society* counterfactual is adopted.

Notes:

(i) The indifference curves in the (*t, G*) space are constructed from the individual utility function $U = U(X,G)$, where X is the sole private good, and the individual budget constraint $X=Y(1- t)$. Substituting the latter into the utility function and totally differentiating we obtain the equation for the slope of the indifference curves in (t, G) space $dt / dG = (\frac{\partial U}{\partial G} / \frac{\partial U}{\partial X}) / Y = (MRS_{GX}) / Y \quad > 0$

(ii) In case of simple proportional taxation (linear income taxation with no lump sum component), the point (t,G) must lie on the dotted tax-share line. With $a \neq 0$, the relationship between the tax rate and the individual tax share is more complicated.

Table 1
Definitions of Coercion
(*Individual-in-society* counterfactual)

	Type of coercion constraint		
Coercion defined with respect to:	**Counterfactual**	***Individual (j = 1...N)***	***Aggregate***
Utility	*Utility if the individual could adjust the level of the public good at the prevailing tax-price*	***Case 1:*** $(V^*_j - V_j) \leq K_j$	***Case 2:*** $\sum_j (V^*_j - V_j) \leq K$
Level of public good	*Desired level of the public good given the prevailing tax-price.*	***Case 3:*** $(G^*_j - G)^2 \leq K_j$	***Case 4:*** $(G^*_j - G)^2 \leq K$

Legend:
G^*_j level of the public good that the individual would like the community to provide at his given tax price.
G actual level of the public good provided.
K_j the degree of coercion for citizen j. We note that the Greek word for coercion is *katanagasmos*.
K *(unsubscripted)*: an aggregate level of coercion.
n_i the number of taxpayer/citizens of type i.
P marginal cost of the public good (assumed constant).
V^*_j maximum desired utility at the individual's given tax price if that person could determine the level of the public good.
V_j actual level of utility.

Figure 2
Coercion Measured by the Level of the Public Good

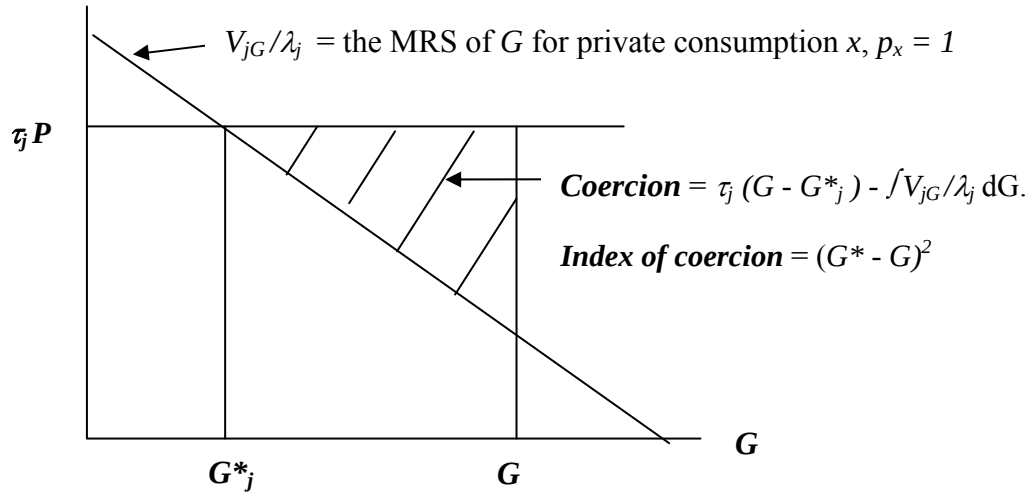
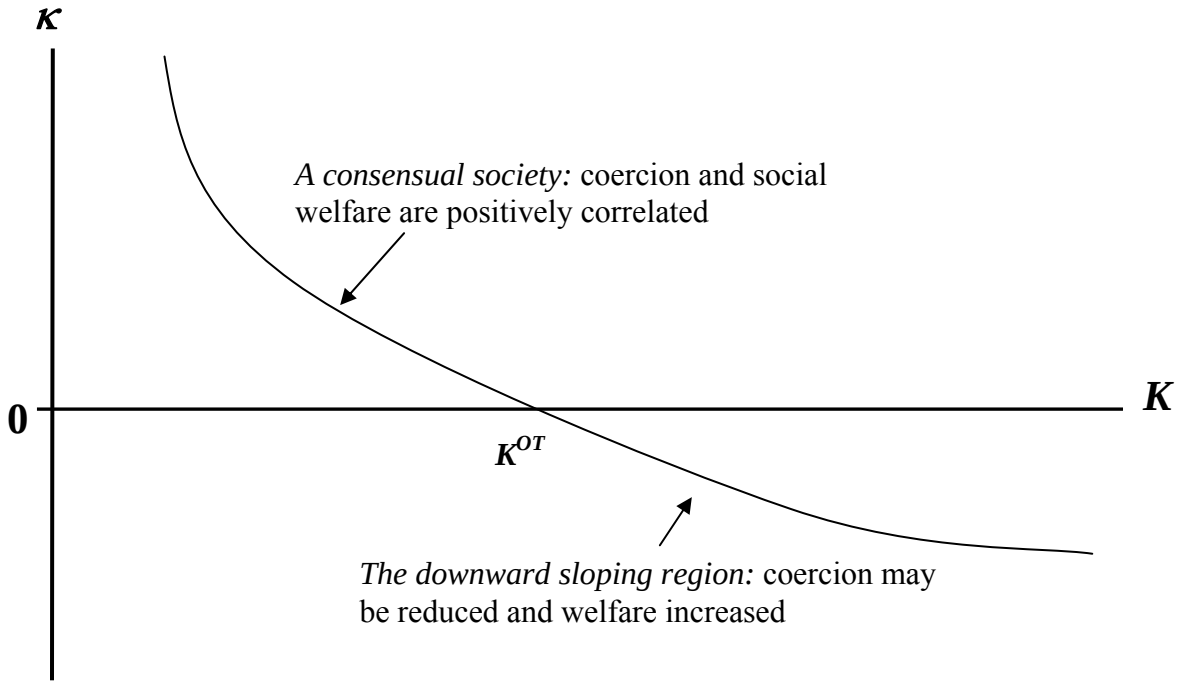
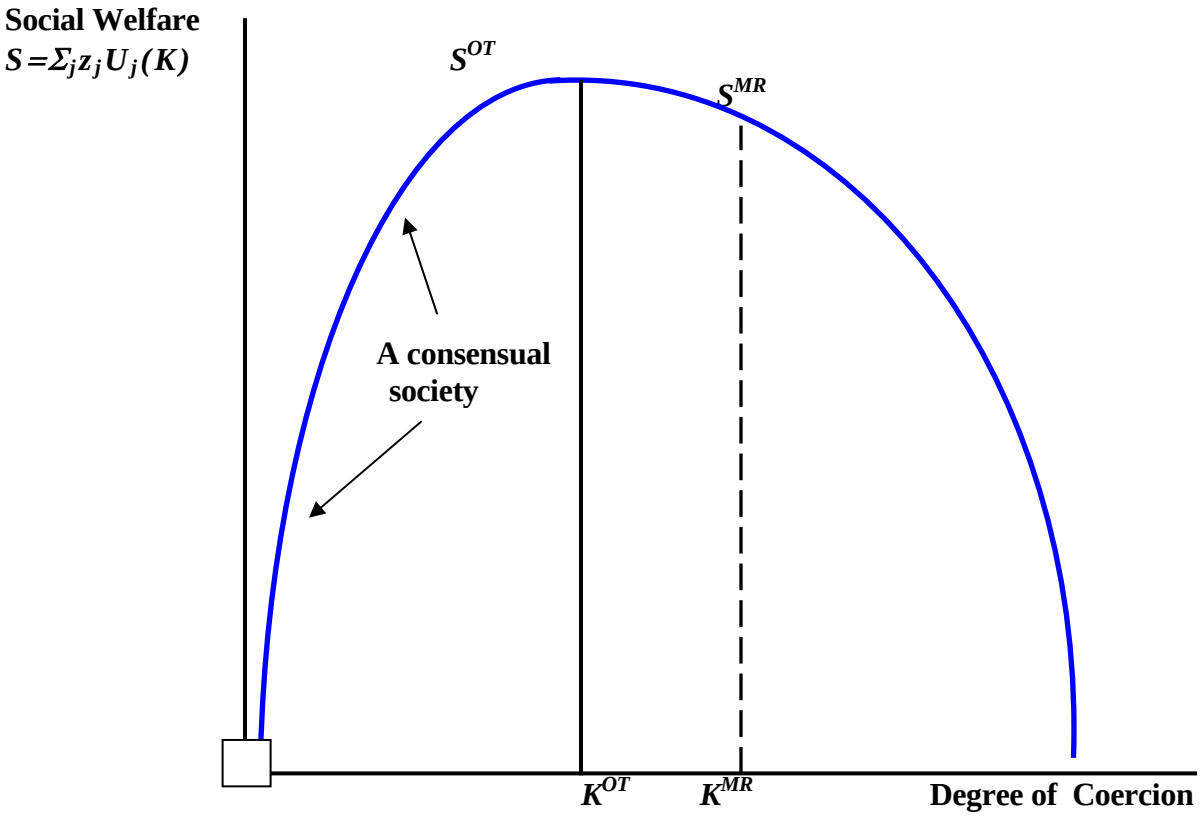


Figure 3
A Possible Relationship Between the Shadow Price of Coercion κ and the Aggregate Degree of Coercion K



K^{OT} = the degree of aggregate coercion corresponding to the traditional (coercion – unconstrained) social plan

Figure 4
The Welfare - Coercion Trade-off and
A Comparison of Social Planning with A Competitive Political Equilibrium



At the origin $K=0$, because the Lindahl solution is not feasible, the planner's coercion-constrained social welfare planning problem is not defined.

K^{OT}: Degree of coercion corresponding to social planning - optimal tax solution
K^{MR}: Degree of coercion corresponding to majority rule in a competitive political system